# Quantum Coulomb Glass on the Bethe Lattice

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# Localization transition of fermions in the presence of disorder

Interacting systems with long-range Coulomb interactions: **Coulomb Glass**

## Peculiar properties of interacting systems

Reduced screening ➡ Anomalies already on *metallic* side

Depleted density of states at Fermi energy on *insulating* side

Obeying Efros-Shklovskii scaling $\rho(\varepsilon) \sim \varepsilon^{d-1}$ (dimension)

**Coulomb Gap**

Glassy dynamics, memory effects...

Efros & Shklovskii (1975), Amir et al. (2011)...

## Classical limit of interacting systems: **Spin glasses**

Insight from (numerically) exactly solvable mean field model ➡ **Sherrington-Kirkpatrick model** $H = -\sum_{i<j} J_{ij} S_i S_j$ ($J_{ij}$: Gaussian random variables)

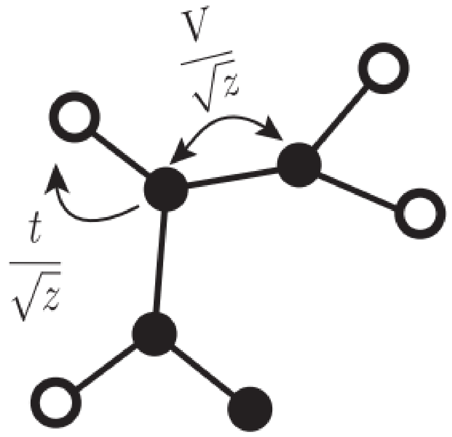
- Low T glassy phase, freezed spins: Edwards – Anderson order parameter $q_{EA} = \overline{m_i^2}$, $m_i = \langle S_i \rangle_H$
- Analogue of Coulomb gap: 'pseudogap' in the distribution local effective magnetic field $h_i = \sum_{j:j\neq i} J_{ij} m_j$ ➡ $P(h) \sim h$
- Glass melted by thermal fluctuations

**Quantum Coulomb Glass?**

**Role of Quantum Fluctuations?**

# Mean Field Electron Glass Model

Disordered interacting fermionic system on a Bethe lattice with coordination # $z \to \infty$




$$\hat{H} = -\frac{t}{\sqrt{z}} \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \mathrm{h.c}) + \frac{V}{\sqrt{z}} \delta\hat{n}_i \delta\hat{n}_j + \sum_i \varepsilon_i \delta\hat{n}_i$$

$\hat{c}_i^\dagger \hat{c}_i - 1/2$

random on-site potential

coordination number

hopping

nearest neighbour repulsion

Random site energies with Gaussian distribution $P(\varepsilon) \sim e^{-\varepsilon^2/(2W^2)}$

A. A. Pastor and V. Dobrosavljević, Phys. Rev. Lett. **83**, 4642 (1999).

Emerging Glassy Phase at Low T

'Structural' transition governed by interactions ⬅ Even in the absence of disorder W=0

Exact mapping to a single site DMFT becomes exact

Mean Field Model for Coulomb Glass ⬌ Sherrington-Kirkpatrick mean-field model for spin glasses

# Quantum Solution

Most previous studies: - Classical spin glass limit ($t = 0$)
- Vicinity of phase transition (Landau theory)

Our Work: Quantum Solution Deep in Glassy Phase

Numerical Ingredients:

- Disorder averaging with replica trick
- Exact mapping to single site problem
- Effective local action:
  a.) continuous time quantum Monte Carlo (CTQMC)
  exact but expensive: only in high-T phase
  b.) iterated perturbation theory (IPT)
  access to glassy phase

Cavity approach:

sum over nn sites of 0

$$S = \int_\tau \bar{c}_{0\,\tau}(\partial_\tau + \epsilon_0)c_{0\,\tau} + \frac{V}{\sqrt{z}}\sum_i{}' \int_\tau \delta n_{0,\tau}\delta n_{i,\tau} - \frac{t}{\sqrt{z}}\sum_i{}' \int_\tau (\bar{c}_{0\,\tau}c_{i\,\tau} + h.c.) + S_{i\neq 0}$$

Integrate out all sites except 0 ➡

$$S_0^{\text{eff}} = \int_\tau \bar{c}_{0\,\tau}(\partial_\tau + \epsilon_0)c_{0\,\tau} - t^2 \int_\tau \int_{\tau'} \bar{c}_{0\,\tau}c_{0\,\tau'} \frac{1}{z}\sum_i{}' \langle c_{i\,\tau}\bar{c}_{i\,\tau'}\rangle_{\text{cav}} + \frac{V}{\sqrt{z}}\sum_i{}' \int_\tau \delta n_{0,\tau}\langle\delta n_{i,\tau}\rangle_{\text{cav}}$$
$$- \frac{V^2}{2}\int_\tau\int_{\tau'} \delta n_{0,\tau}\delta n_{0,\tau'} \frac{1}{z}\sum_i{}' \langle\delta n_{i,\tau}\delta n_{i,\tau'}\rangle_{\text{cav}} + \ldots$$

cavity averages in the absence of 0

➡ $$S_0^{\text{eff}} = \int_\tau\int_{\tau'} \left\{\bar{c}_{0\,\tau}\left(\delta(\tau-\tau')(\partial_\tau + \tilde{\epsilon}_0) - t^2 G(\tau-\tau')\right)c_{0\,\tau'} - \frac{V^2}{2}\chi(\tau-\tau')\delta n_{0,\tau}\delta n_{0,\tau'}\right\}$$

Gaussian random field includes Hartree shift

average Green's function

average density correlator

➡ Ensemble of actions + field distribution $\tilde{P}(\tilde{\epsilon})$ : self-consistent solution

Low T: many symmetry broken states, have to include all of them

cavity approach becomes more complicated

→ Replica trick $\overline{F} \sim \overline{\log Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$

**We can still map to problem to an ensemble of effective local actions**

$$S_{\rm loc}(\widetilde{\varepsilon}) = \int_\tau \int_{\tau'} \left\{ \bar{c}_\tau \left[ \delta_{\tau,\tau'} \left[ \partial_{\tau'} + \widetilde{\varepsilon} \right] - t^2\, G(\tau-\tau') \right] c_{\tau'} - \frac{V^2}{2}\, \widetilde{\chi}(\tau-\tau')\, \delta n_\tau\, \delta n_{\tau'} \right\} - \frac{\beta\, \widetilde{\varepsilon}}{2} \quad + \widetilde{P}(\widetilde{\varepsilon})$$

Self-consistency conditions: $\begin{Bmatrix} G(\tau) \\ \widetilde{\chi}(\tau) \end{Bmatrix} = \int d\widetilde{\varepsilon}\, \widetilde{P}(\widetilde{\varepsilon}) \begin{Bmatrix} G_{\widetilde{\varepsilon}}(\tau) \\ \widetilde{\chi}_{\widetilde{\varepsilon}}(\tau) \end{Bmatrix}$ ← $\langle c_\tau\, \bar{c}_{\tau'} \rangle_{\rm loc}$ ; ← $\langle \delta n_\tau\, \delta n_{\tau'} \rangle_{\rm loc} - \langle \delta n_\tau \rangle^2_{\rm loc}$ both calculated from $S_{\rm loc}(\widetilde{\varepsilon})$

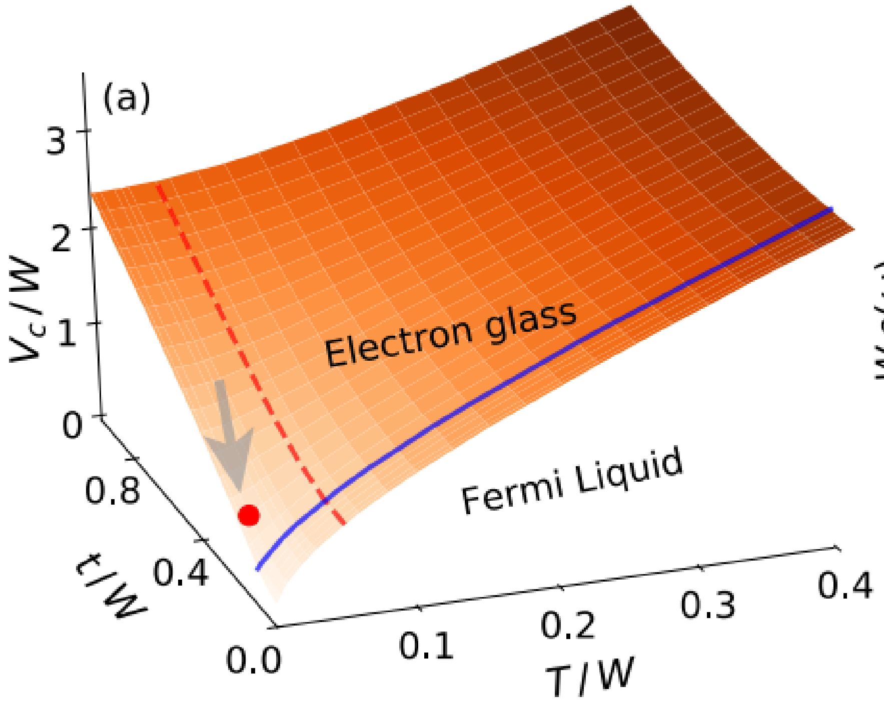
**Electron glass transition**

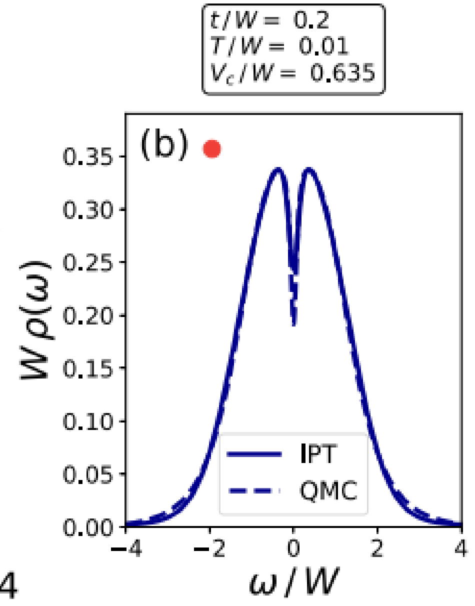
$\widetilde{P}(\widetilde{\varepsilon})$ : - **Gaussian** in high T phase

Variance renormalized by interactions

- **Pseudo-gap** behaviour in glassy phase

*Glass melted by thermal or quantum fluctuations*



Phase diagram: - continuous time quantum Monte Carlo (CTQMC)

- iterated perturbation theory (IPT)

## CTQMC

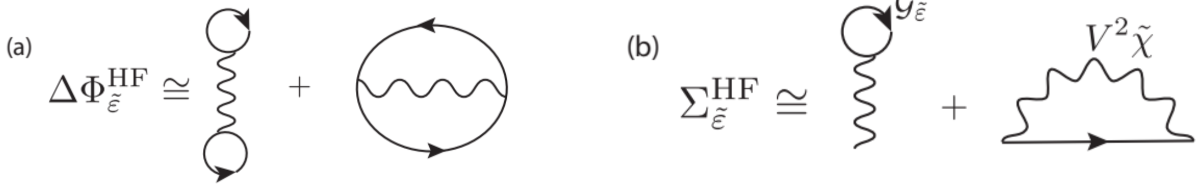
Expand partition function in hybridization $t^2\, G(\tau - \tau')$

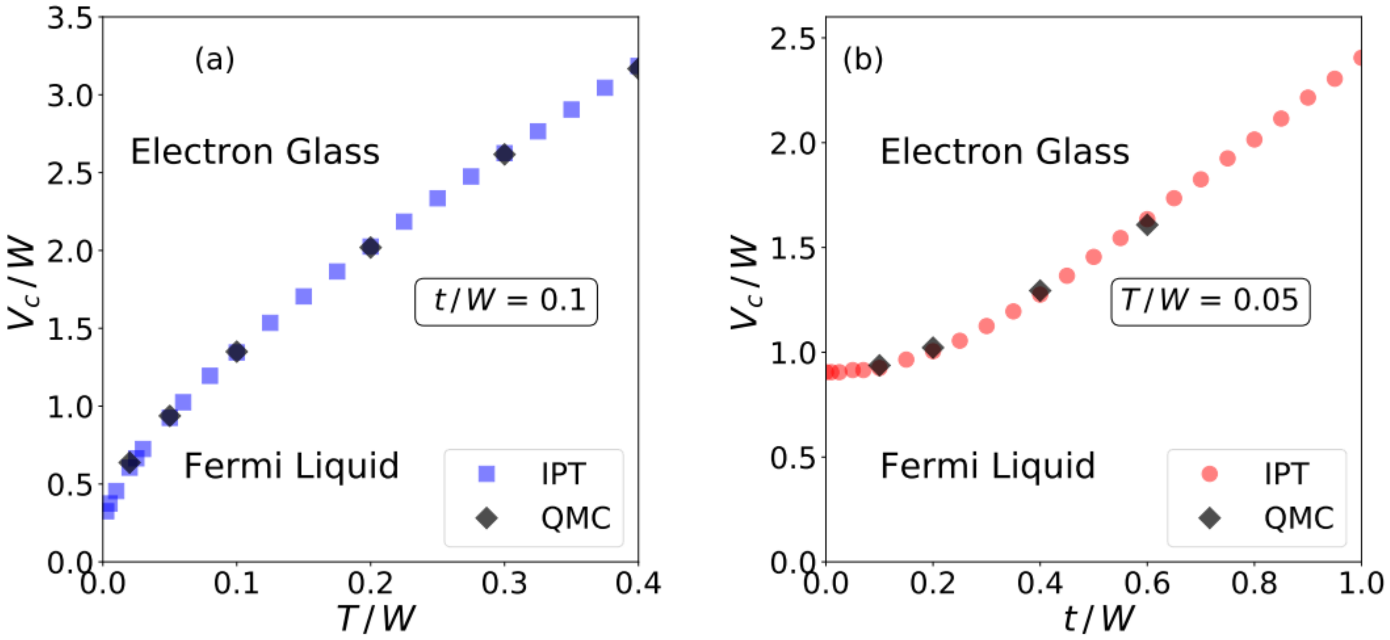
→ Set of diagrams for observables

Stochastic sampling of diagrams

## IPT

First order expansion in $V^2\, \tilde{\chi}(\tau - \tau') \equiv V^2\, (\chi(\tau - \tau') - \langle \delta n \rangle^2)$



# Glassy phase

(IPT results)

# Pseudo-gap opening

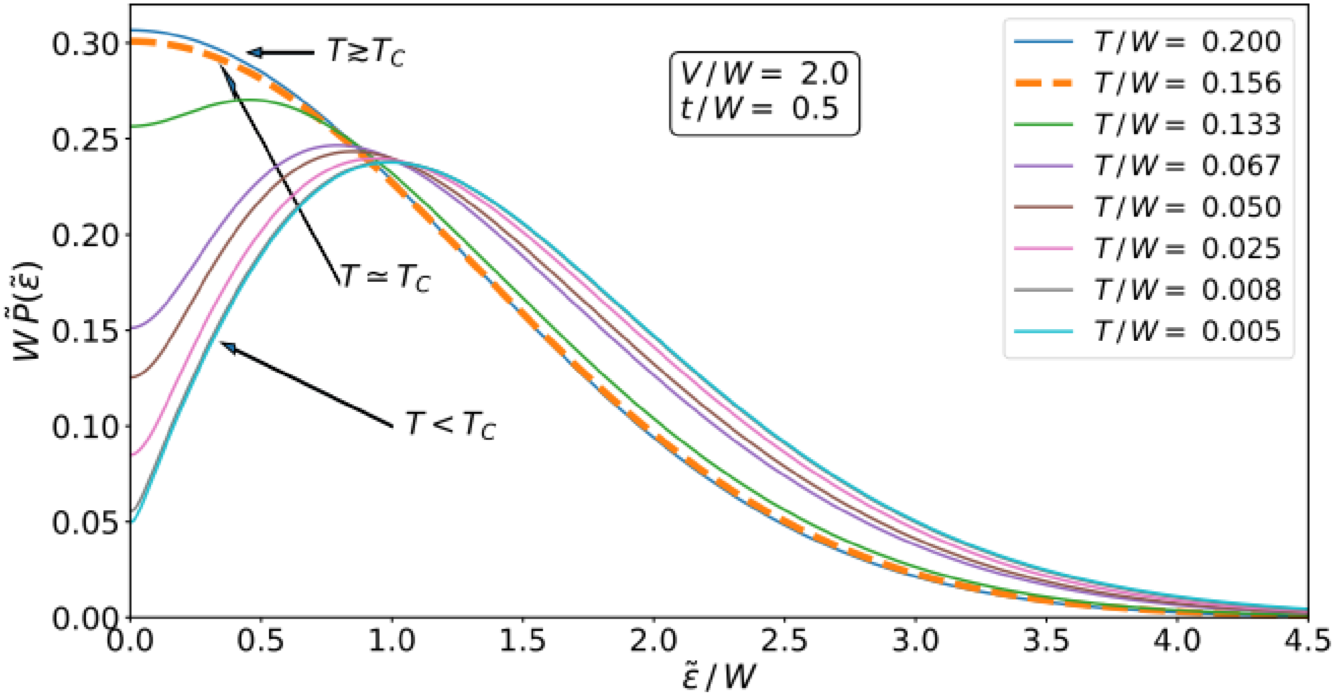
Glass Transition Driven by Thermal or Quantum Fluctuations

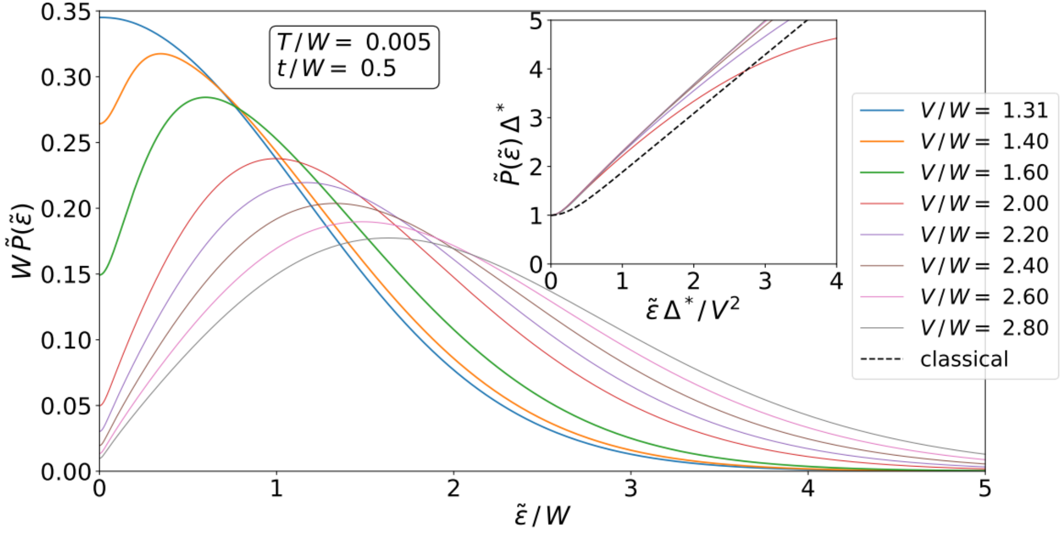
Efros-Shklovskii Scaling
Deep in Glassy Phase

$$\widetilde{P}\left(\widetilde{\varepsilon}\right) \approx 1.13\ |\widetilde{\varepsilon}|/V^2$$

⇕

Sherrington-Kirkpatrick spin glass result




*Universal regime determined by interactions*

⇕

*Structural transition driven by interactions alone*

# Pseudo-Gap in Spectral Function

$$\rho(\omega) = \frac{1}{\pi}\,\mathrm{Im}G(\omega + i\,0^{+})$$ ⟺ Average Local Density of States Accessible in Tunneling Experiments

Efros-Shklovskii Scaling

$$\rho(\omega) \propto \omega/V^2$$

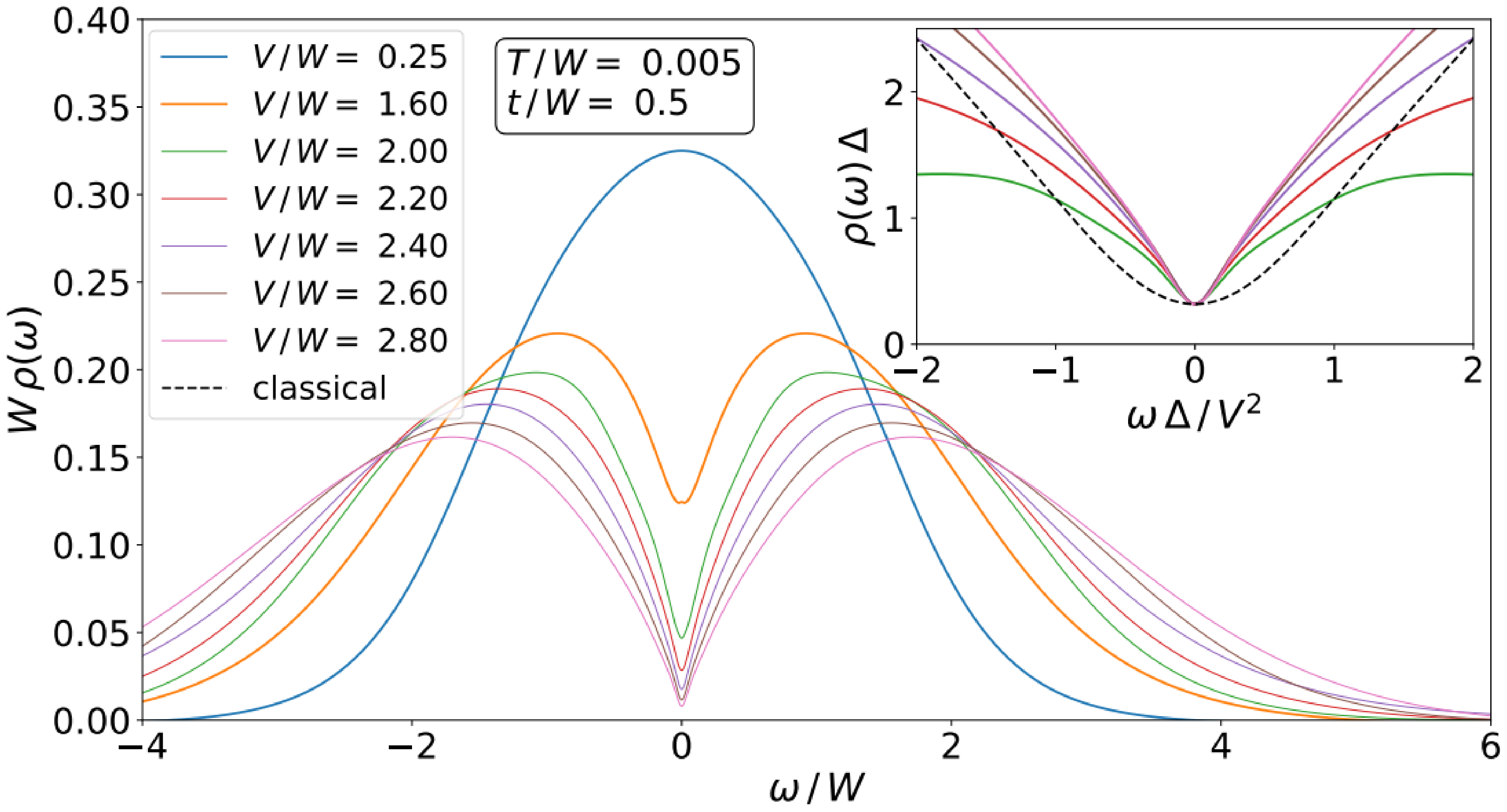
Quantum Scaling Function

≠

Classical Scaling Function

⬇

Slight Difference Between Quantum and Thermal Fluctuations



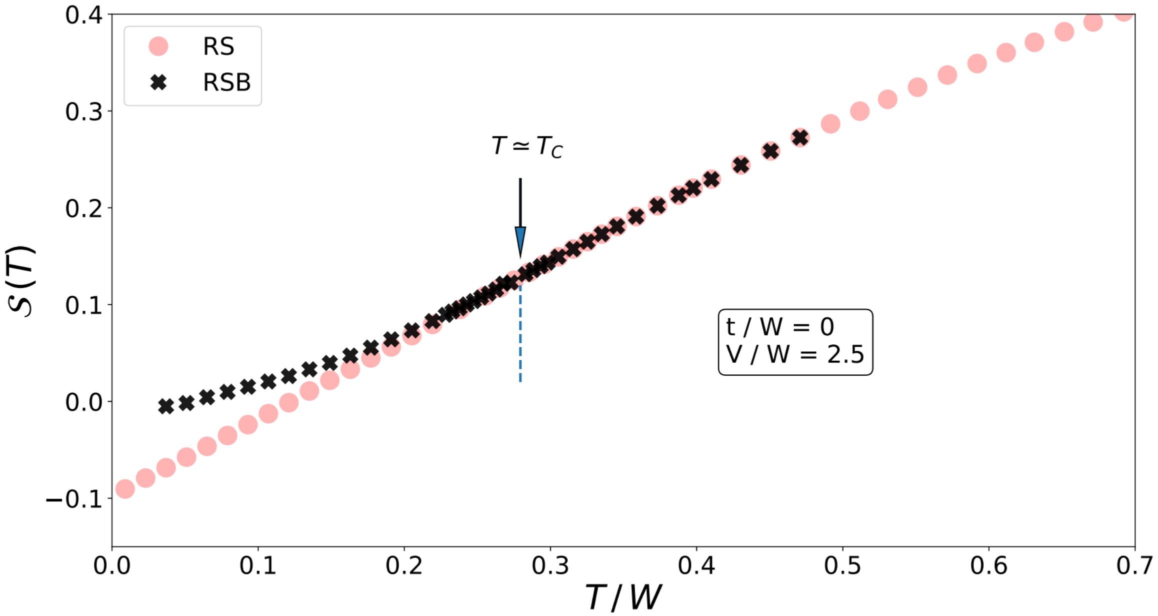
# Thermodynamics

Order of phase transition? $\mathcal{S} = \frac{\partial \Phi_{\text{latt}}}{\partial T}$

High precision required, not very reliable because of perturbative approach + numerical errors

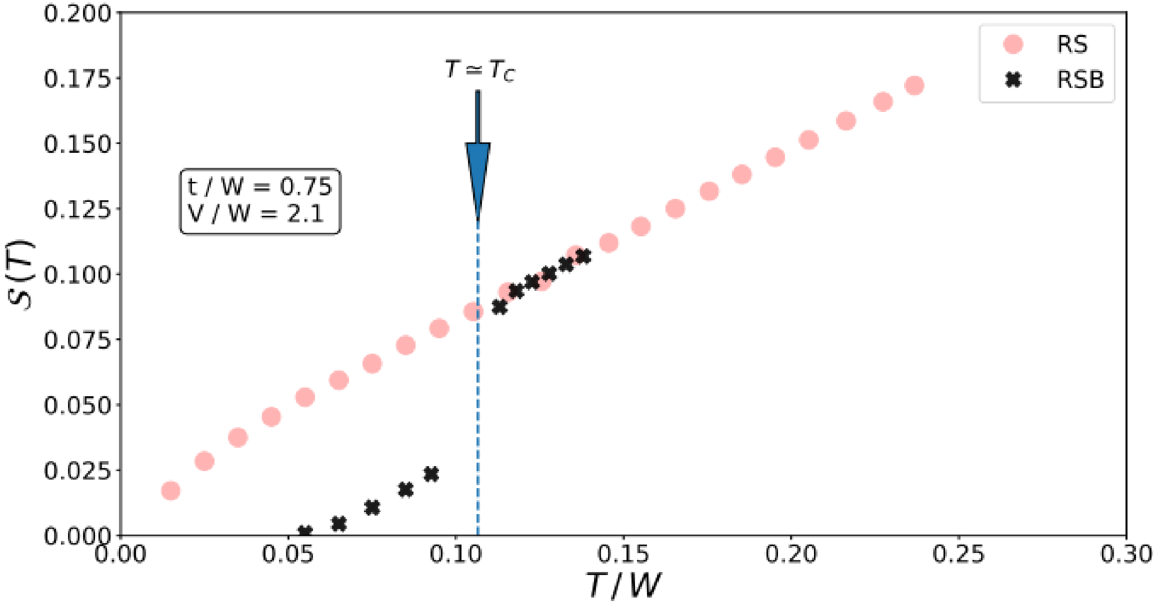
## Classical case: (at least) second order transition

Fermi liquid solution unphysical at low T: negative entropy




## Quantum case: First order transition

Problem with previous calculations based on Landau approach?



# Summary

**Investigation of a mean field electron glass model in the quantum regime**

Phase diagram

Properties of the glassy phase (full replica symmetry breaking)

- Pseudogap in spectral function and in distribution of effective Hartree fields
- Universal regime: Efros-Shklovkii scaling
- First order transition away from classical limit (?)

*Example for structural transition, governed by interactions (not disorder)*

# Outlook / Open questions

→ Numerically exact solution with CTQMC in glassy phase: work in progress

→ Role of spin degrees of freedom

→ Coulomb glass in 2D/3D?

→ Dynamics, anomalously slow response

Thank you for your attention!