Fractional topology from entanglement

Joel Hutchinson, Karyn Le Hur

arXiv:2002.11823

CPHT Centre de Physique Théorique

cnrs

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Outline

- Motivation
- Ising entanglement model
- Generalized spin models
- Lattice model

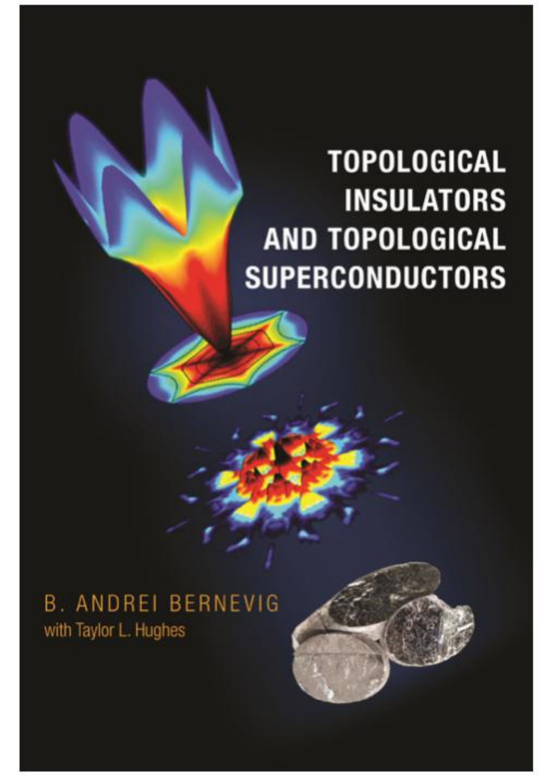
Motivation

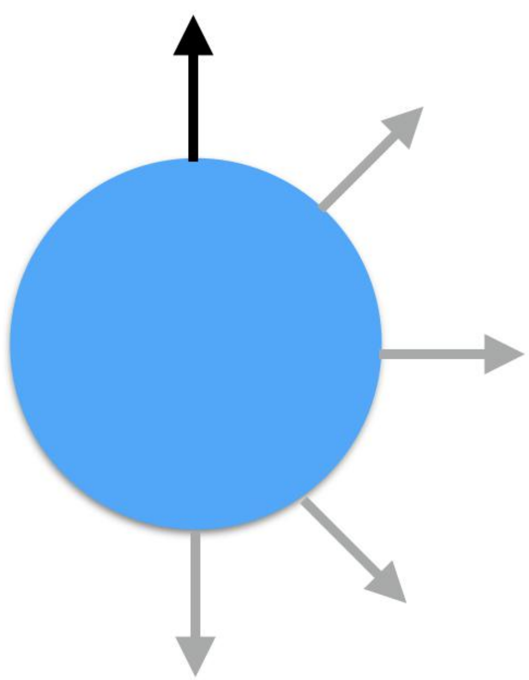
- Simple example of topology:
  spin-1/2 in radial magnetic field.

$$\vec{H} = H(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$|\psi_-\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} \quad |\psi_+\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

ground state





Motivation

- Simple example of topology:
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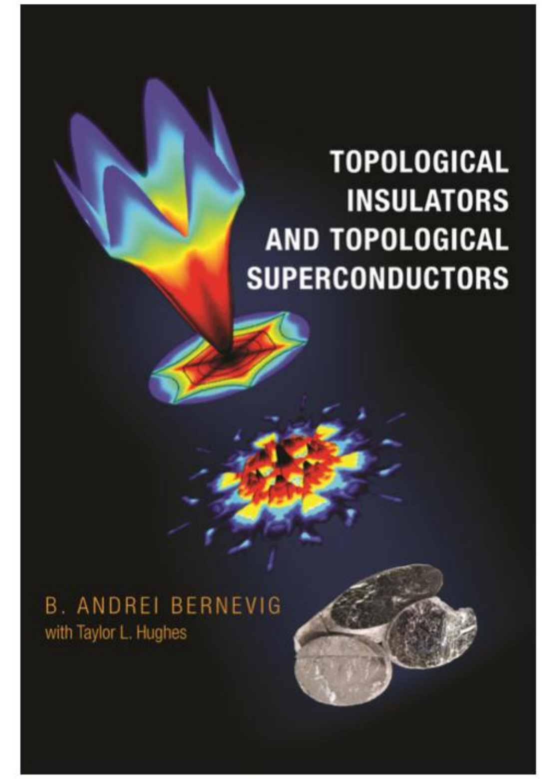
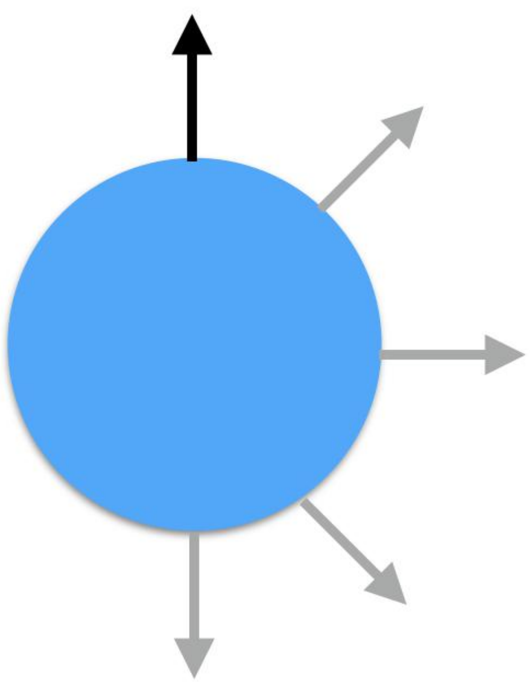


$$\vec{H} = H(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$|\psi_-\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} \quad |\psi_+\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

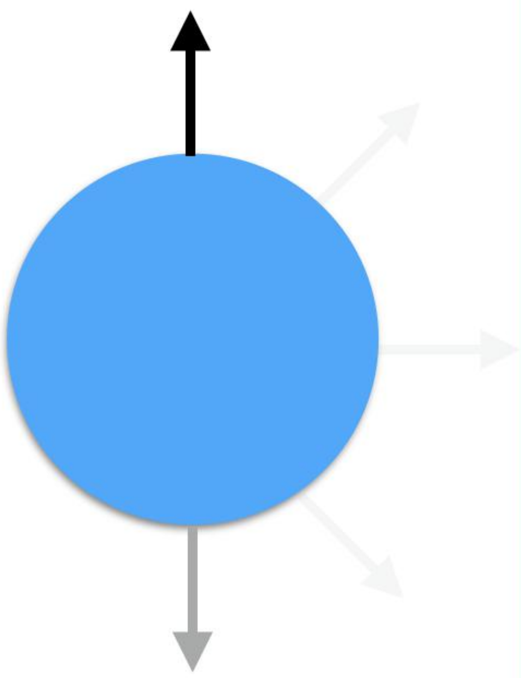
ground state



- Berry connection: $\mathcal{A}_\alpha = i\langle\psi|\partial_\alpha|\psi\rangle$
- Berry curvature: $\mathcal{F}_{\phi\theta} = \partial_\phi\mathcal{A}_\theta - \partial_\theta\mathcal{A}_\phi = \dfrac{\sin\theta}{2}$
- Chern number: $\mathcal{C} = \dfrac{1}{2\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \mathcal{F}_{\phi\theta}$

$$= 1$$

Motivation

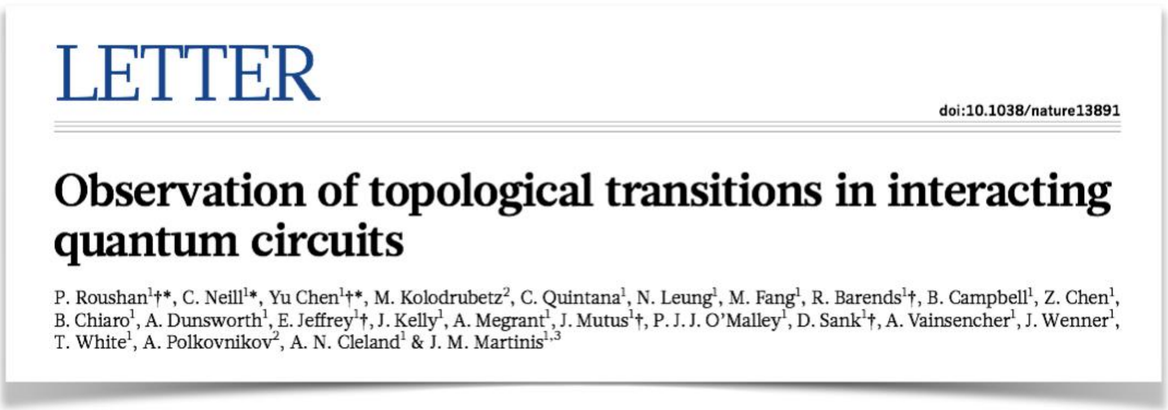


- One can measure topology from time-evolution of qubits from North to South poles.

$$\mathcal{C} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \mathcal{F}_{\phi\theta}$$
$$= \frac{1}{2}\Big(\langle\sigma_z(\theta=0)\rangle - \langle\sigma_z(\theta=\pi)\rangle\Big)$$

L. Henriet, A. Sclocchi, P. P. Orth, and K. Le Hur, Phys. Rev. B **95**, 054307 (2017).

- Works for Two qubits as well.



LETTER

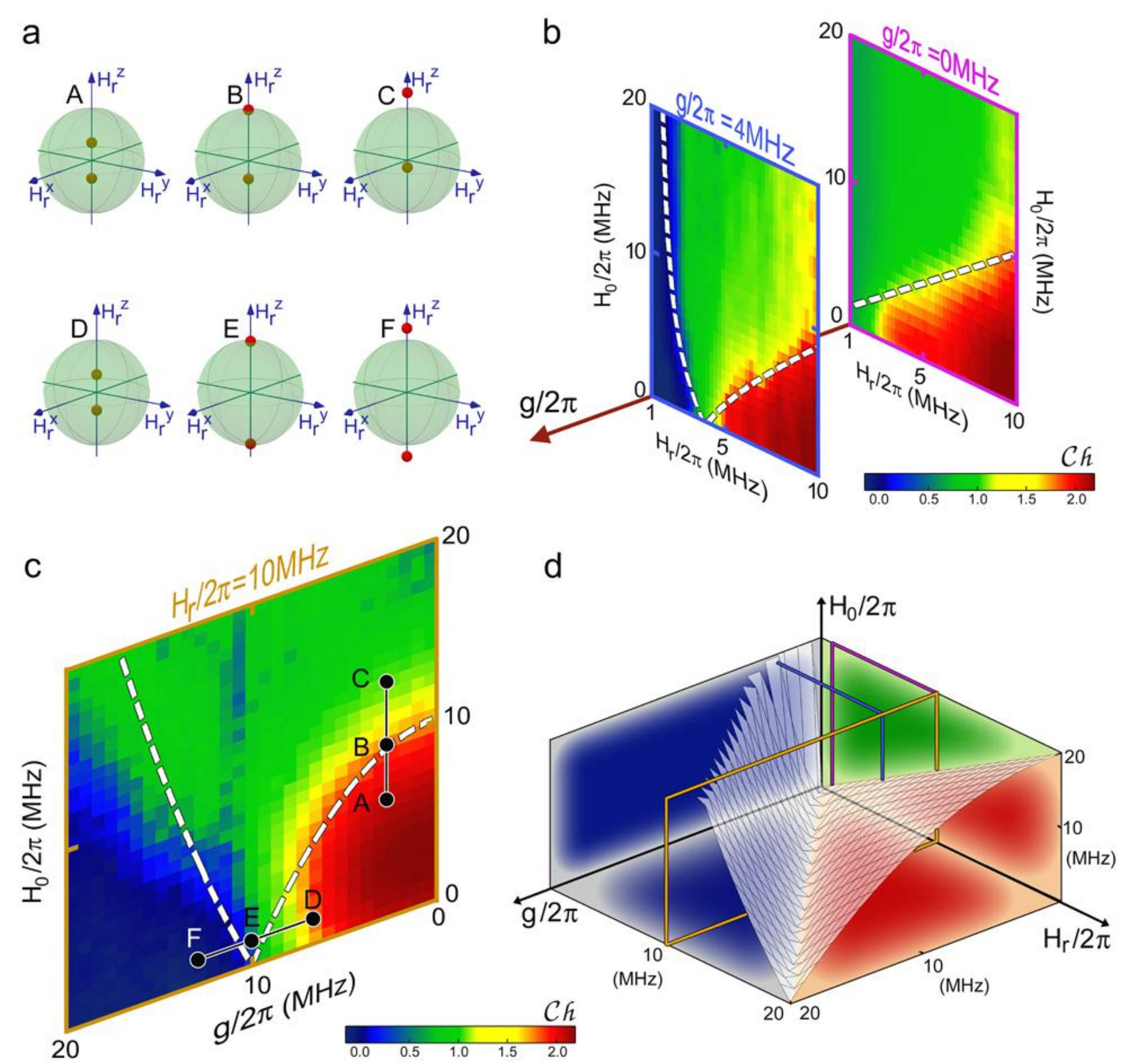
doi:10.1038/nature13891

Observation of topological transitions in interacting quantum circuits

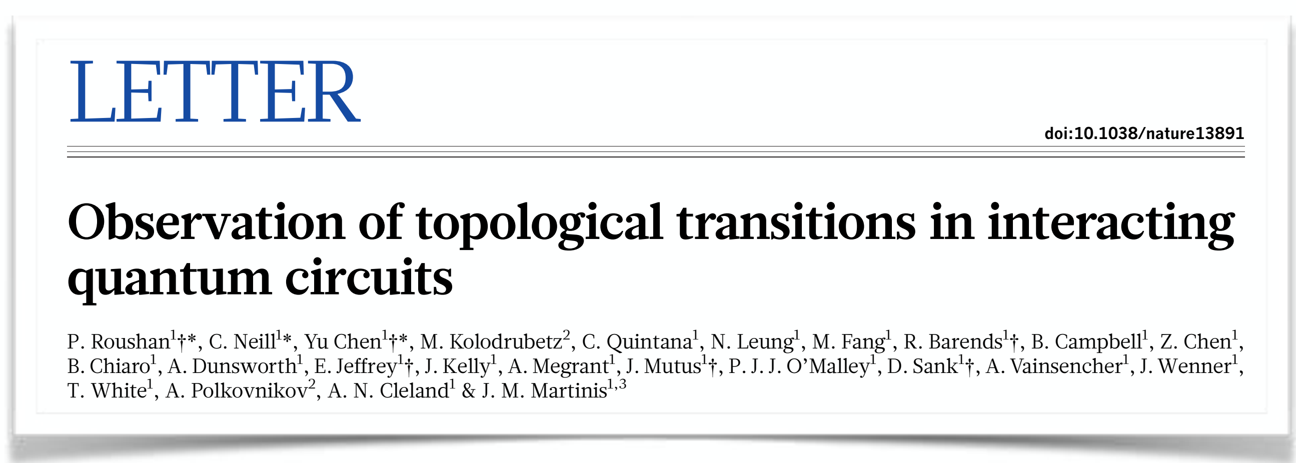
P. Roushan[1]†*, C. Neill[1]*, Yu Chen[1]†*, M. Kolodrubetz[2], C. Quintana[1], N. Leung[1], M. Fang[1], R. Barends[1]†, B. Campbell[1], Z. Chen[1], B. Chiaro[1], A. Dunsworth[1], E. Jeffrey[1]†, J. Kelly[1], A. Megrant[1], J. Mutus[1]†, P. J. J. O'Malley[1], D. Sank[1]†, A. Vainsencher[1], J. Wenner[1], T. White[1], A. Polkovnikov[2], A. N. Cleland[1] & J. M. Martinis[1,3]

Motivation

- Total Chern number 0,1,2 measured experimentally.



- Can be interpreted as number of degeneracy monopoles contained in parameter sphere.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Motivation

- Does there exist a Chern number for each spin?

Yes!

- well-defined.
- gauge invariant.
- robust to symmetry-preserving deformations.

$$\mathcal{A}_\alpha = i\langle\psi|\partial_\alpha|\psi\rangle \longrightarrow \mathcal{A}^j_\alpha \equiv i\langle\psi|\partial^j_\alpha|\psi\rangle$$

$$\begin{aligned}\mathcal{C}^j &= -\frac{1}{2\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \partial^j_\theta \mathcal{A}^j_\phi \\ &= \frac{1}{2}\left(\langle\sigma^j_z(\theta=0)\rangle - \langle\sigma^j_z(\theta=\pi)\rangle\right)\end{aligned}$$

Outline

Motivation

Ising entanglement model

Generalized spin models

Lattice model

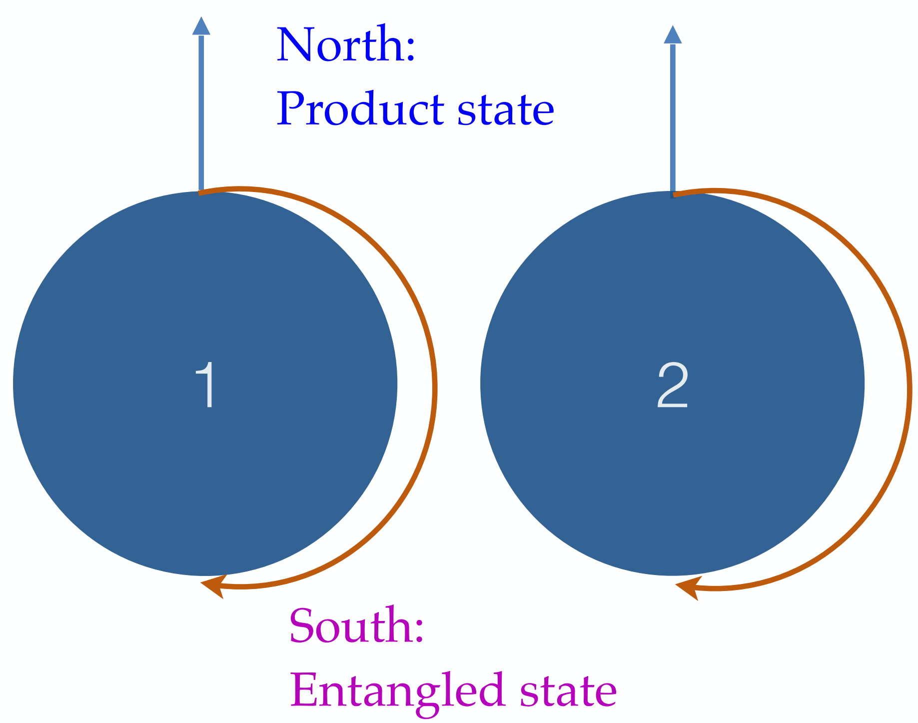
Ising Entanglement Model

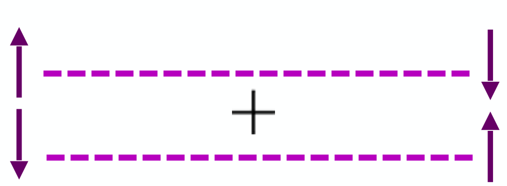
$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2$$

Radial magnetic fields

(angle-dependent) Ising interaction

$$\vec{H} = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, \cos\theta + M)^T$$



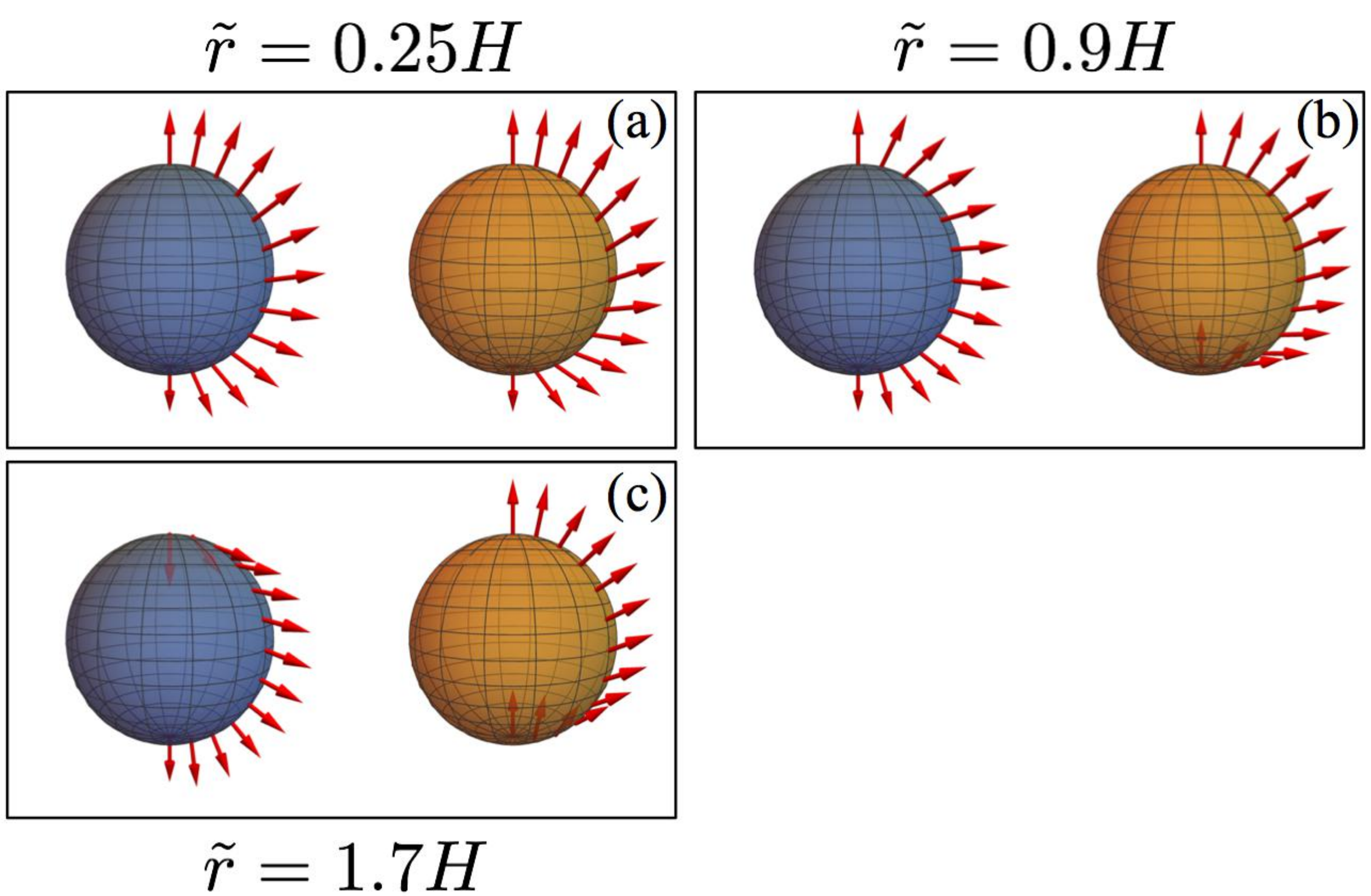


(can also be obtained with interaction that increases with angle)

Ising Entanglement Model

- Time-dependent sweep protocol

$$\theta = vt \qquad \phi = \text{const} \qquad M_1 = \frac{H}{3} \qquad M_2 = \frac{H}{2}$$



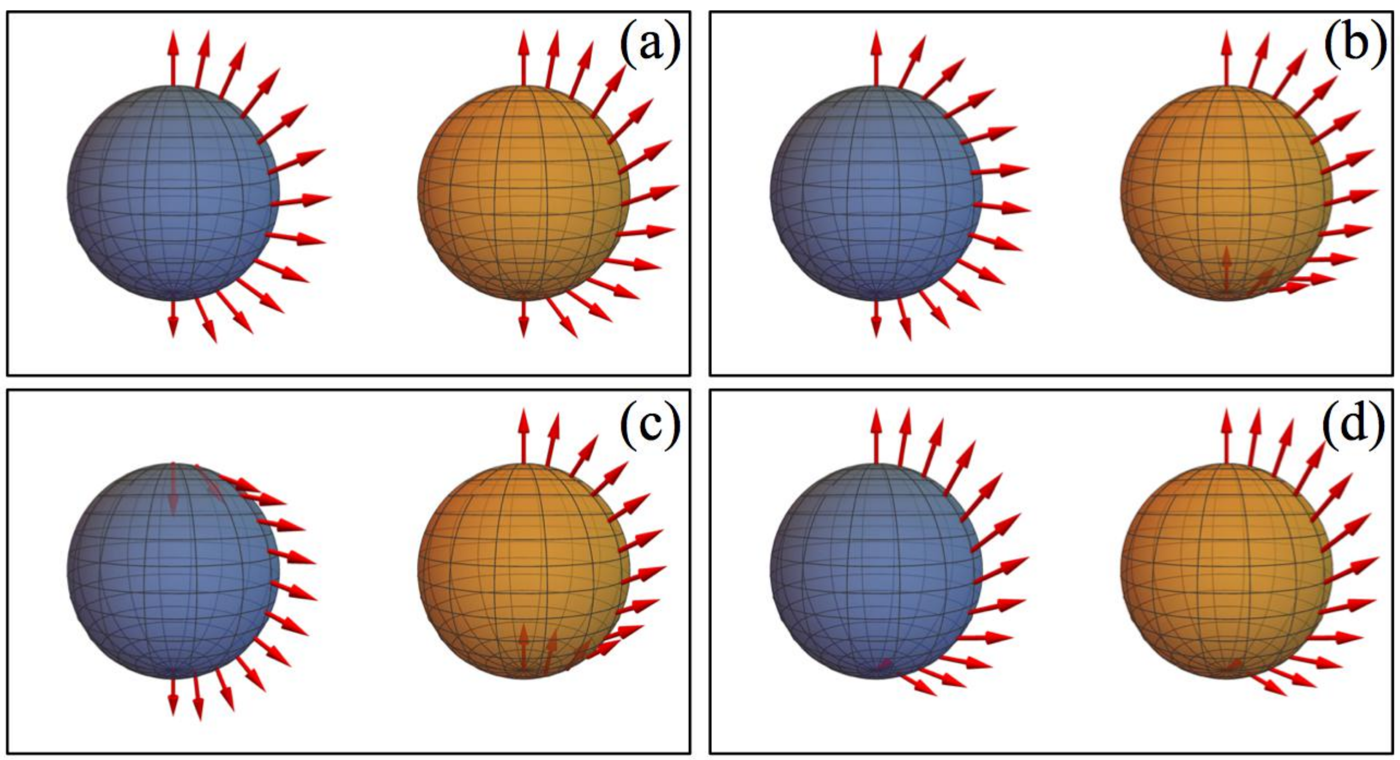
Ising Entanglement Model

- Time-dependent sweep protocol

$$\theta = vt$$
$$\phi = \text{const}$$

$$M_1 = \frac{H}{3} \qquad M_2 = \frac{H}{2}$$

$\tilde{r} = 0.25H$ $\qquad$ $\tilde{r} = 0.9H$

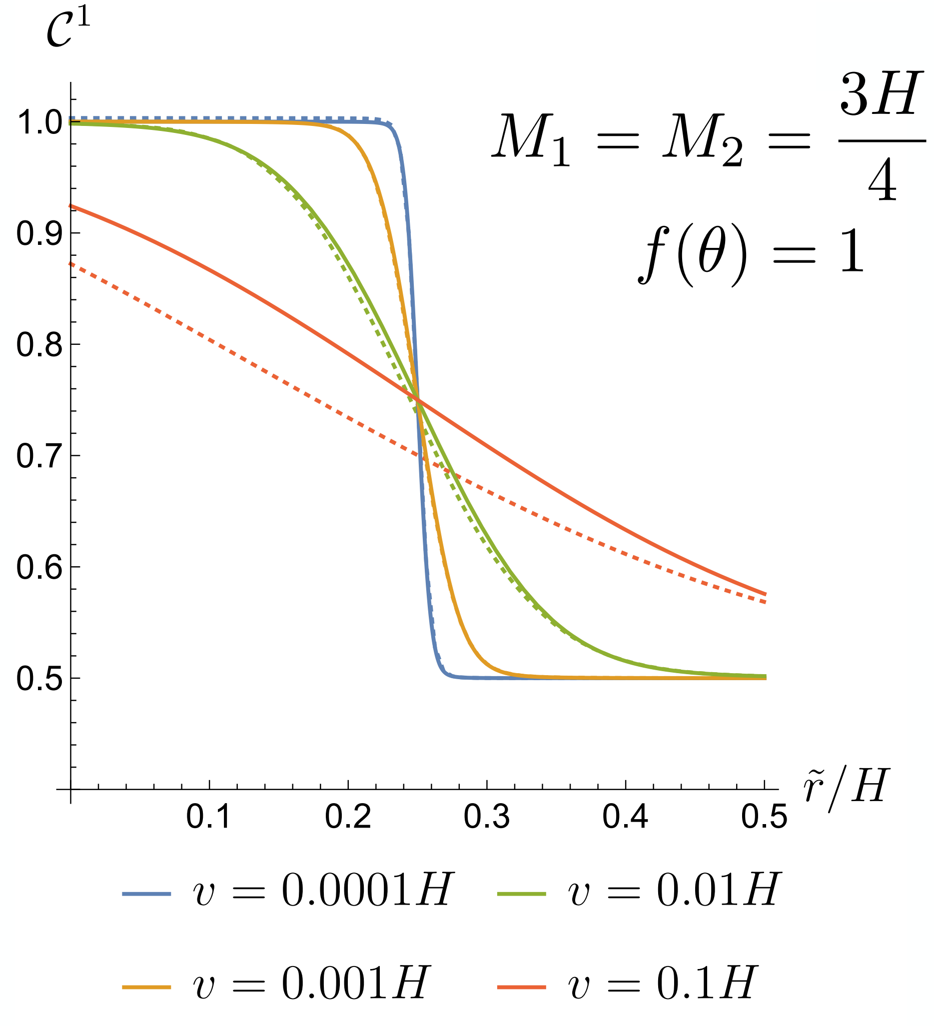


$\tilde{r} = 1.7H$ $\qquad$ $\tilde{r} = 0.33H$

$$M_1 = M_2 = \frac{3H}{4}$$

Ising Entanglement Model

- Access partial Chern numbers from poles



Ising Entanglement Model

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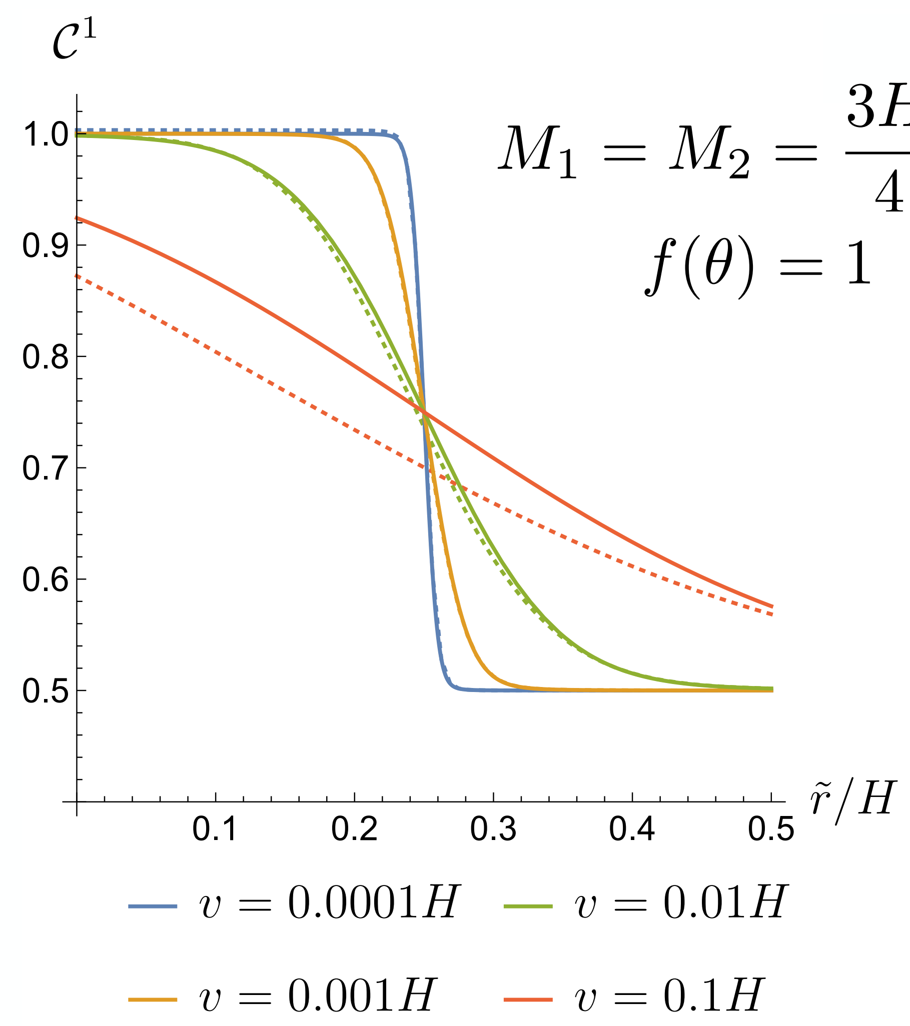
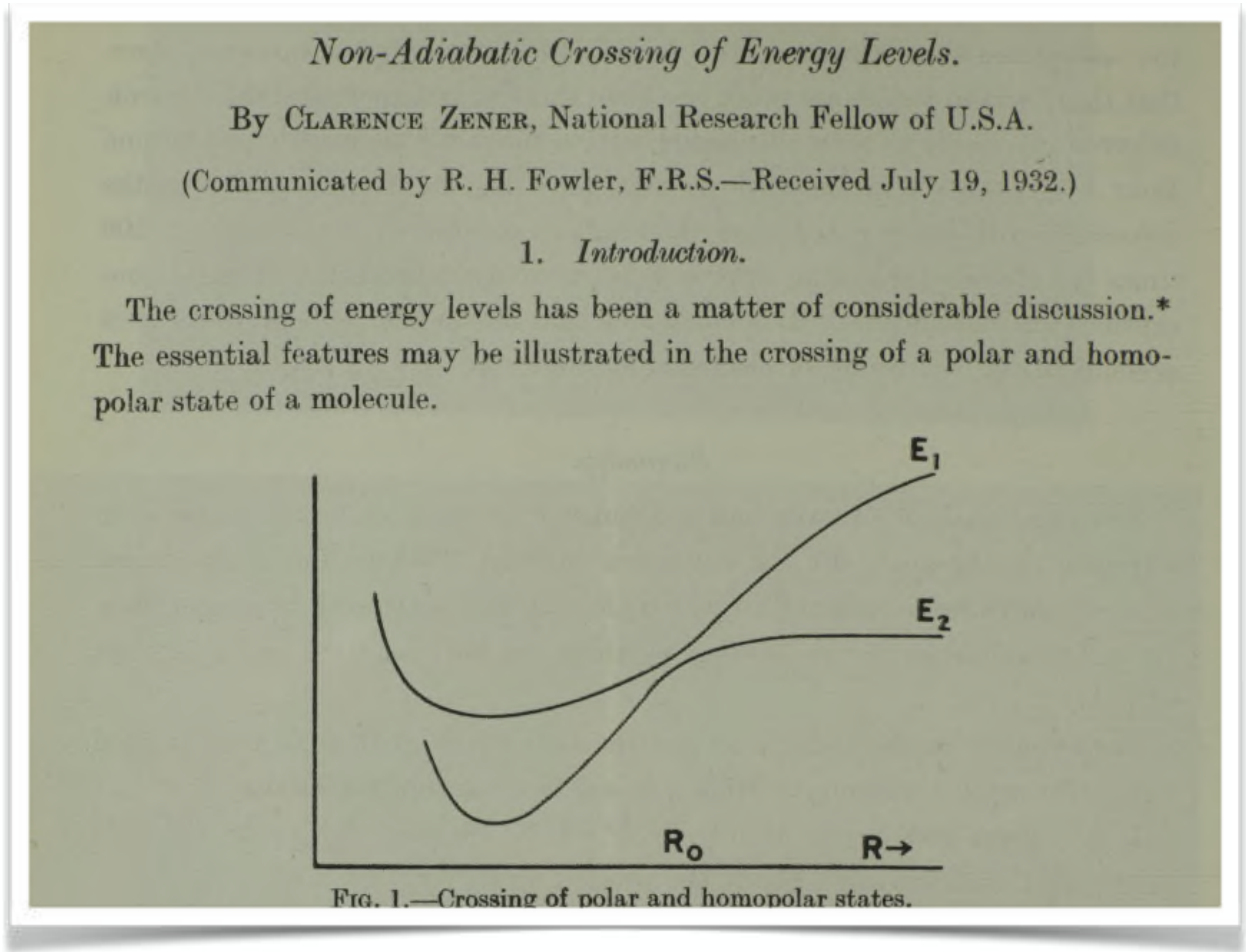


$$M_1 = M_2 = \frac{3H}{4}$$

$$f(\theta) = 1$$

- Landau-Zener diabatic transitions



Non-Adiabatic Crossing of Energy Levels.

By Clarence Zener, National Research Fellow of U.S.A.

(Communicated by R. H. Fowler, F.R.S.—Received July 19, 1932.)

1. *Introduction.*

The crossing of energy levels has been a matter of considerable discussion.* The essential features may be illustrated in the crossing of a polar and homopolar state of a molecule.

Fig. 1.—Crossing of polar and homopolar states.

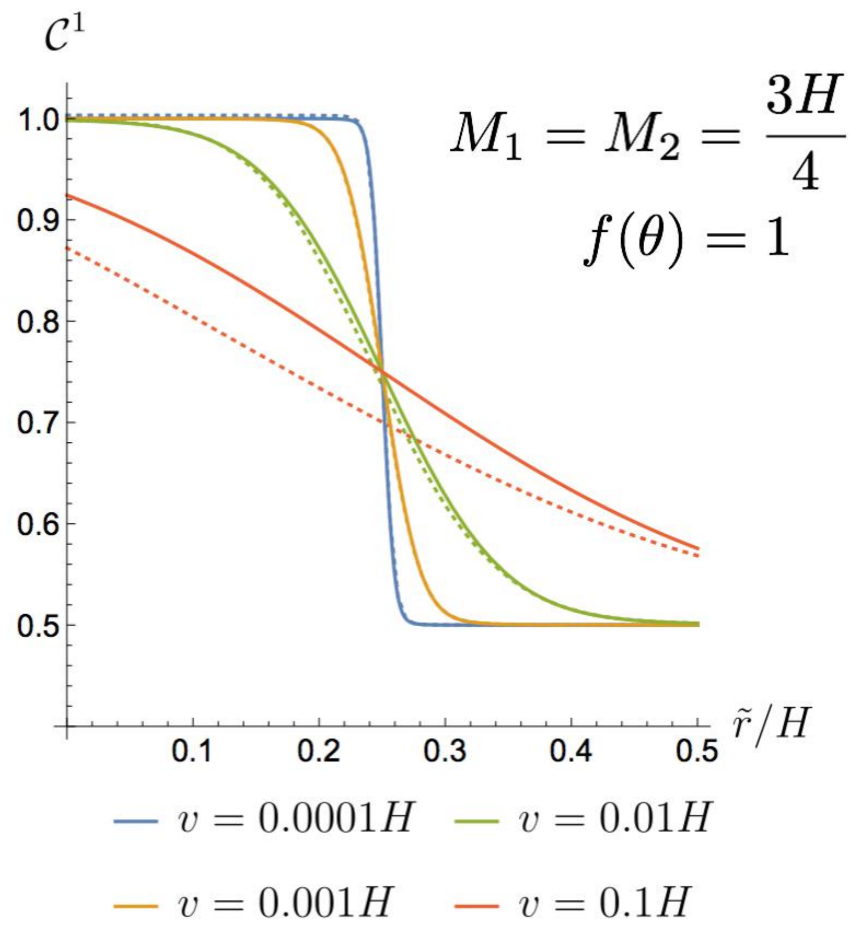
Adiabaticity: $\gamma = \dfrac{\Delta^2}{\sqrt{2}Hv}$

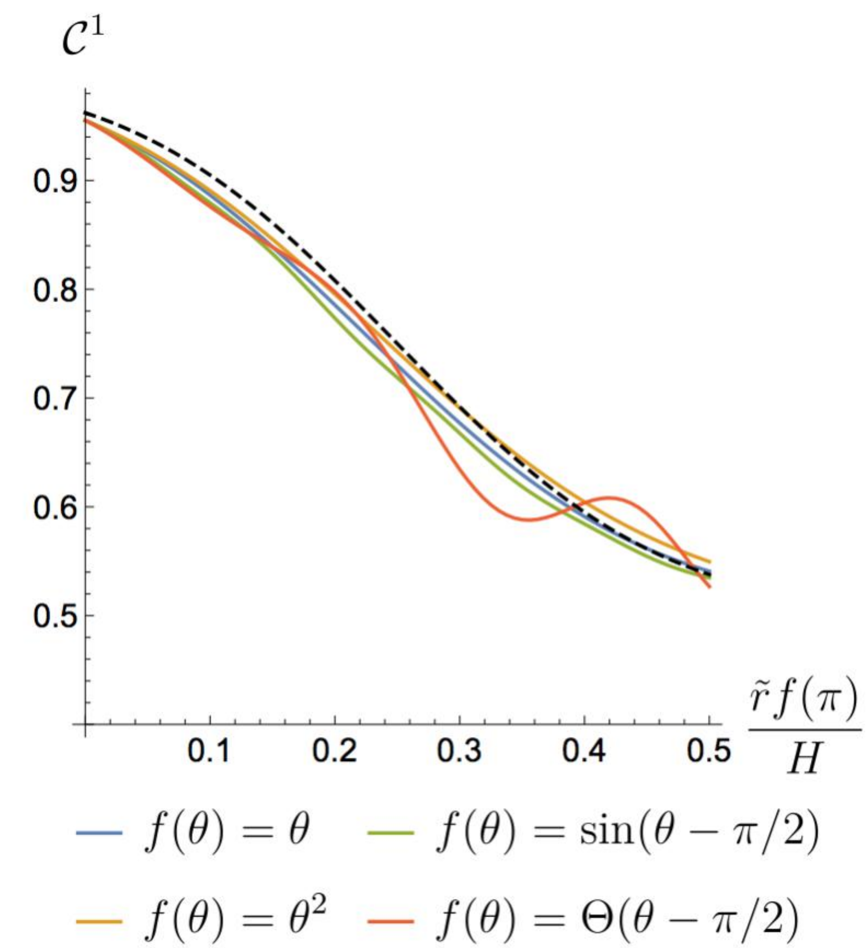
$$\mathcal{C}^j \approx \frac{3}{4} + \frac{\pi}{4}\mathrm{Re}\left(e^{i3\pi/4}e^{-\gamma\pi/4}\frac{\mathrm{sgn}(\Delta)\sqrt{\gamma}}{\Gamma(1/2+i\gamma/4)\Gamma(1-i\gamma/4)}\right)$$

X

Ising Entanglement Model

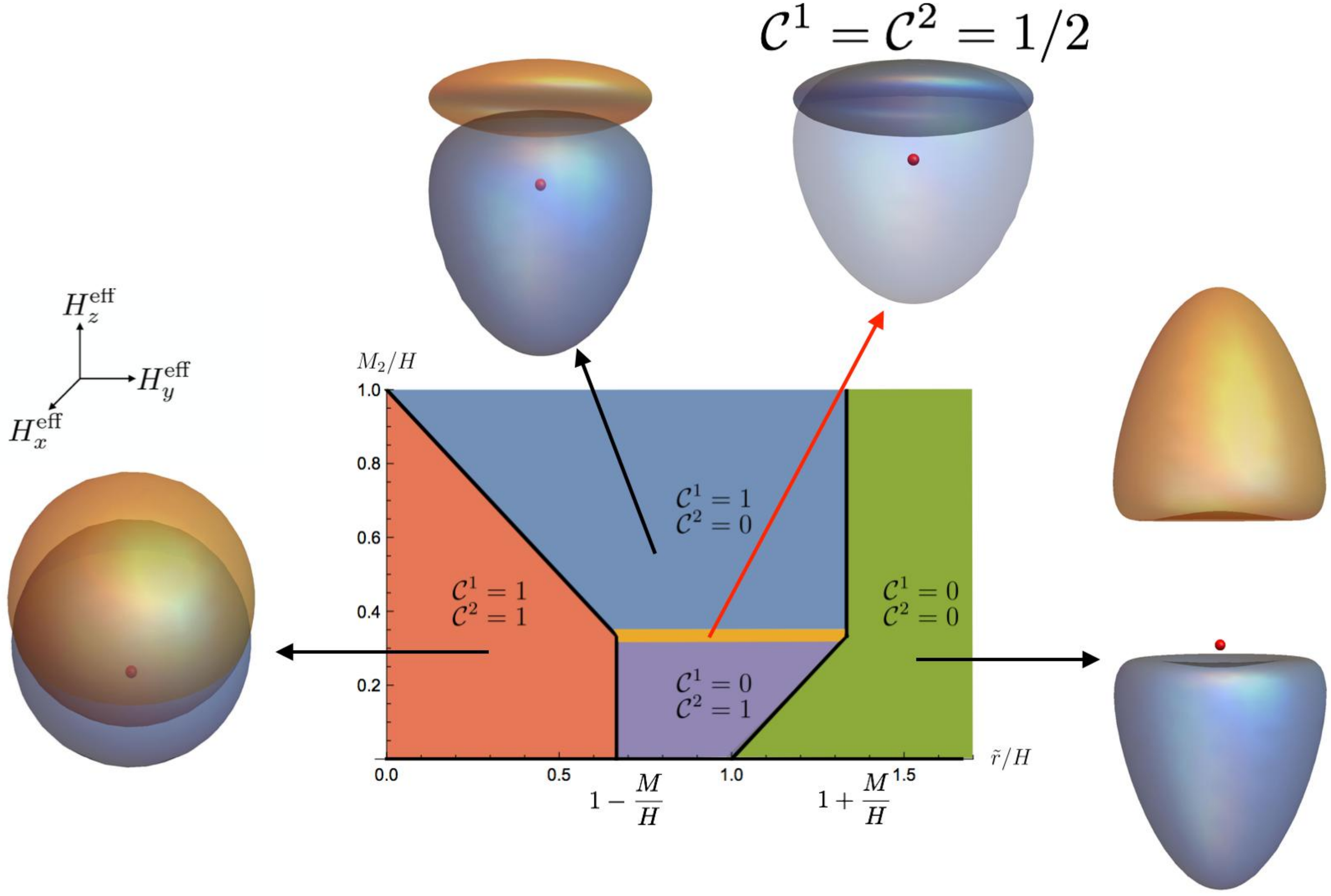
- Access partial Chern numbers from poles





Ising Entanglement Model

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2$$



Outline

Motivation

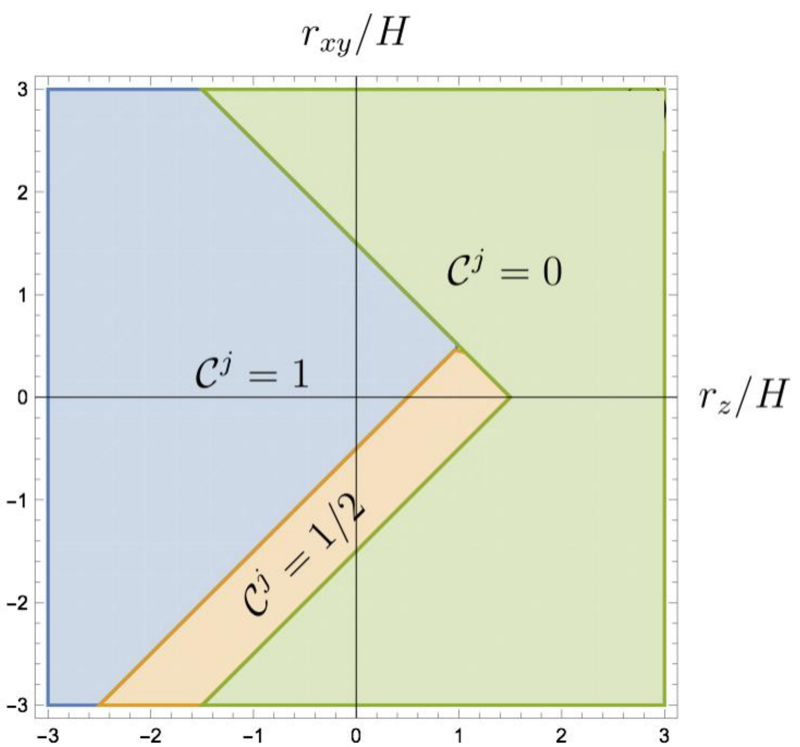
Ising entanglement model

Generalized spin models

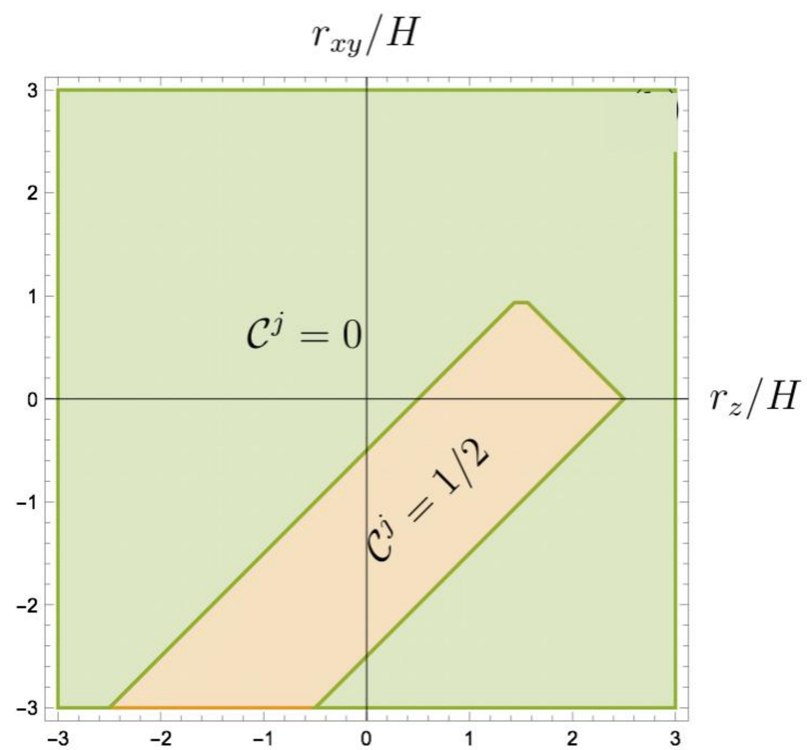
Lattice model

Generalized Interactions

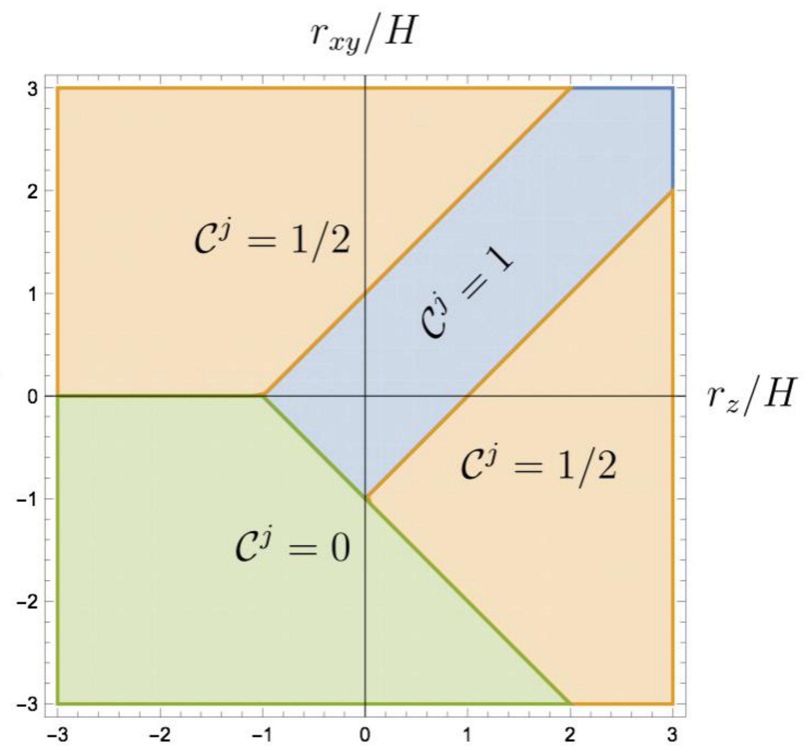
$$\mathcal{H} = -\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}_1 - \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}_2 + r_z \sigma_1^z \sigma_2^z + r_{xy}(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y)$$



$M < H$



$M > H$



$M = 0$

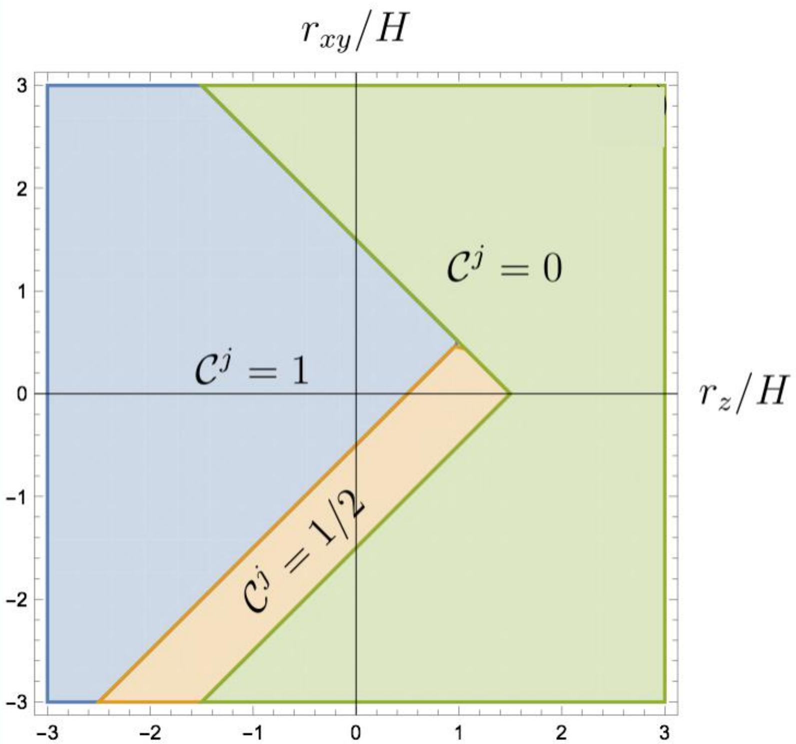
theta-dependent coupling

$r_z \to r_z \sin(\theta - \pi/2)$

$r_{xy} \to r_{xy} \sin(\theta - \pi/2)$

Generalized Interactions

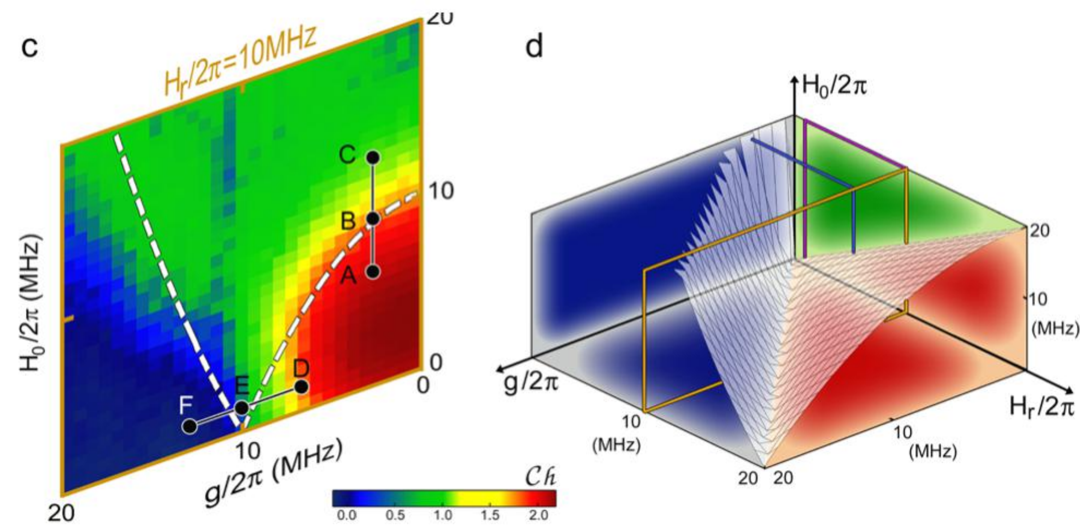
$$\mathcal{H} = -\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}_1 - \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}_2 + r_z \sigma_1^z \sigma_2^z + r_{xy}(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y)$$

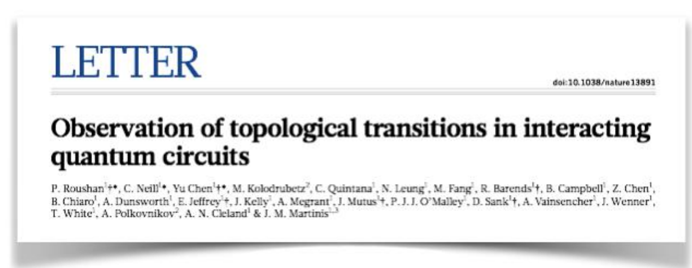


$M < H$

c.f.

$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0 \sigma_1^z + \mathbf{H_1} \cdot \boldsymbol{\sigma_1} + \mathbf{H_2} \cdot \boldsymbol{\sigma_2} - g(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y)]$$





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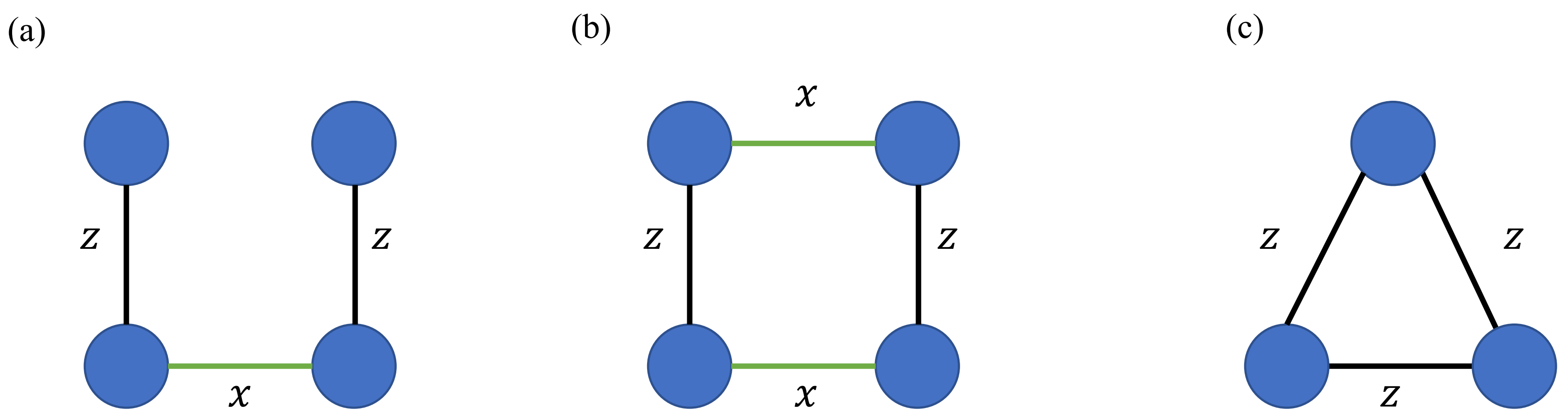
Higher Spins

- Check with 2nd-order perturbation theory to find g.s. at south pole:

$$\mathcal{H}_{\text{eff}} = P\mathcal{H}'P + P\mathcal{H}'\frac{1-P}{E_D - H_0}\mathcal{H}'P$$



$\mathcal{C}^j = \{1,0,0,1\}$ $\mathcal{C}^j = 1/2$ $\mathcal{C}^j = 2/3$

Higher Spins

- Models that admit fractional invariants:
  - 2-spin Ising coupled 1/2
  - 2-spin XY coupled 1/2
  - 2-spin Heisenberg X
  - 2-spin anisotropic Heisenberg 1/2
  - Inversion symmetric X
  - 4-spin ZXZ box X
  - 4-spin ZXZX box 1/2
  - Even-N Ising chain 1/2
  - Odd-N Ising chain (N+1)/2N

Outline

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Ising entanglement model

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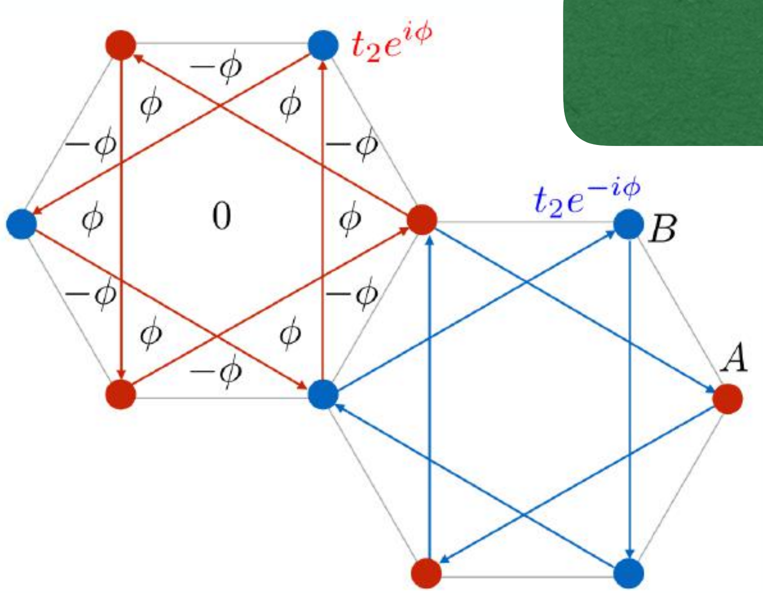
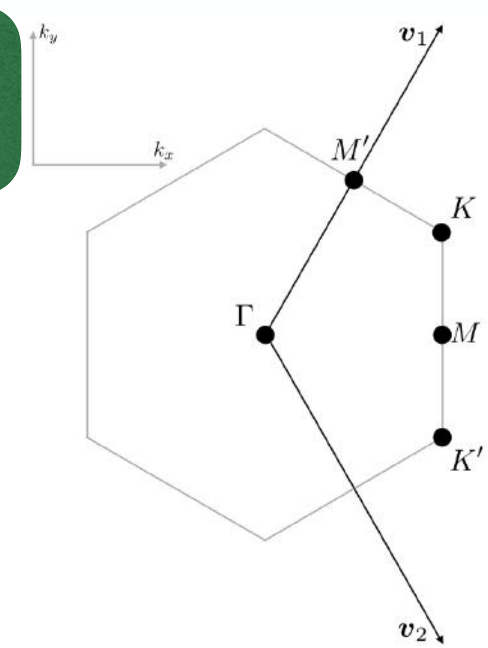
Lattice model





Lattice Model

- Mapping between 2 qubits and bilayer:

Hexagonal lattice	Spheres
1st BZ	S^2
K, K'	N, S
Haldane hopping elements	Radial magnetic field
Bilayer (2 flavours)	2 spins
$\mathcal{C}$	$\frac{1}{2}(\langle\sigma^z(\theta=0) - \sigma^z(\theta=\pi))$
$n_{\mathbf{k}A} - n_{\mathbf{k}B}$	σ^z
Semenoff mass	Offset magnetic field
density-density interaction	Ising interaction

Lattice Model

- Mapping between 2 qubits and bilayer:

Hexagonal lattice

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} (\boldsymbol{d} + M_1\hat{z}) \cdot \boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\boldsymbol{d} + M_2\hat{z}) \cdot \boldsymbol{\sigma} \end{pmatrix}$$

Spheres

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2$$

↑

interlayer hopping (in perturbation theory)

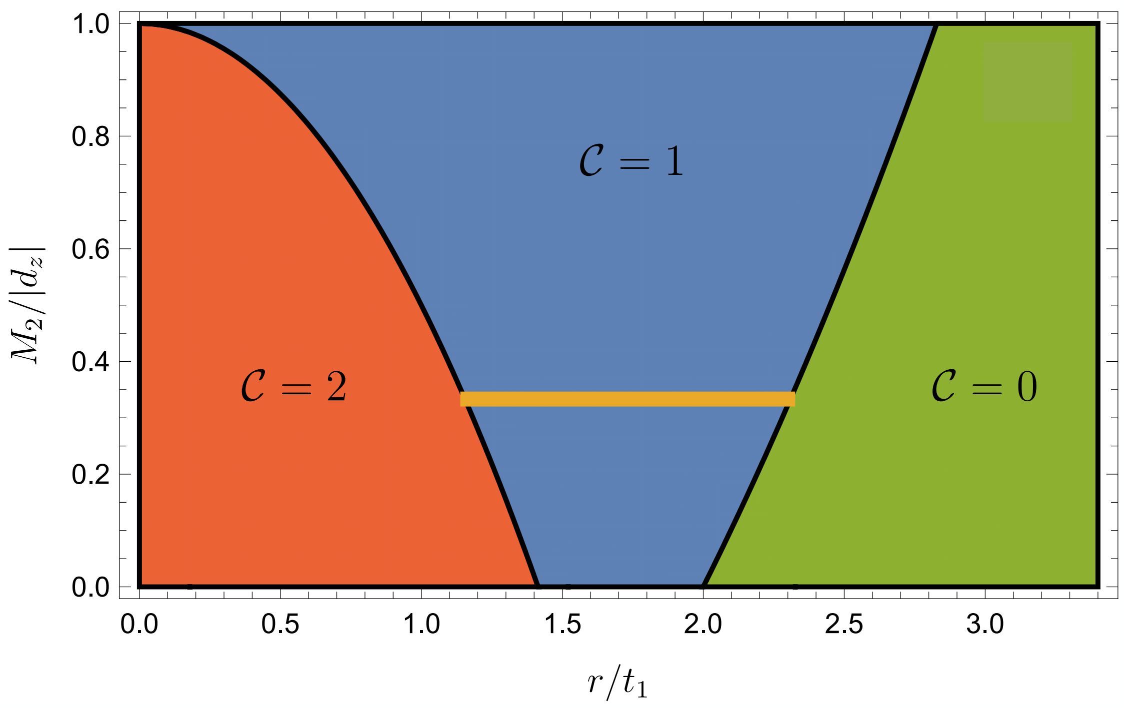
↑

density-density interaction	Ising interaction

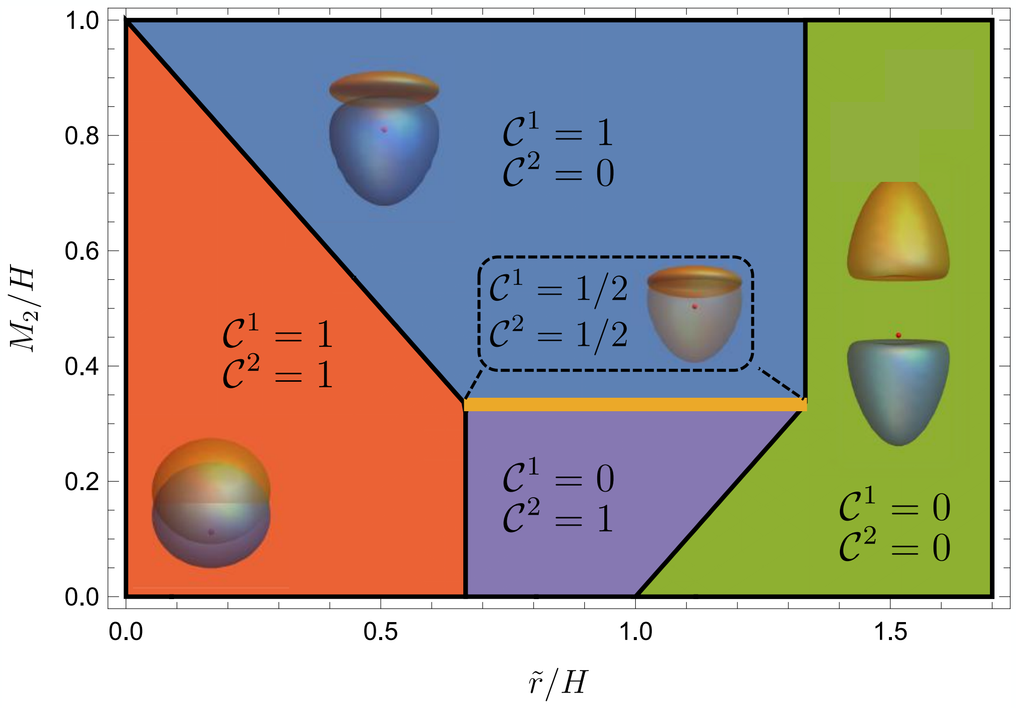
Lattice Model

- Compute the total Chern number:

Hexagonal lattice

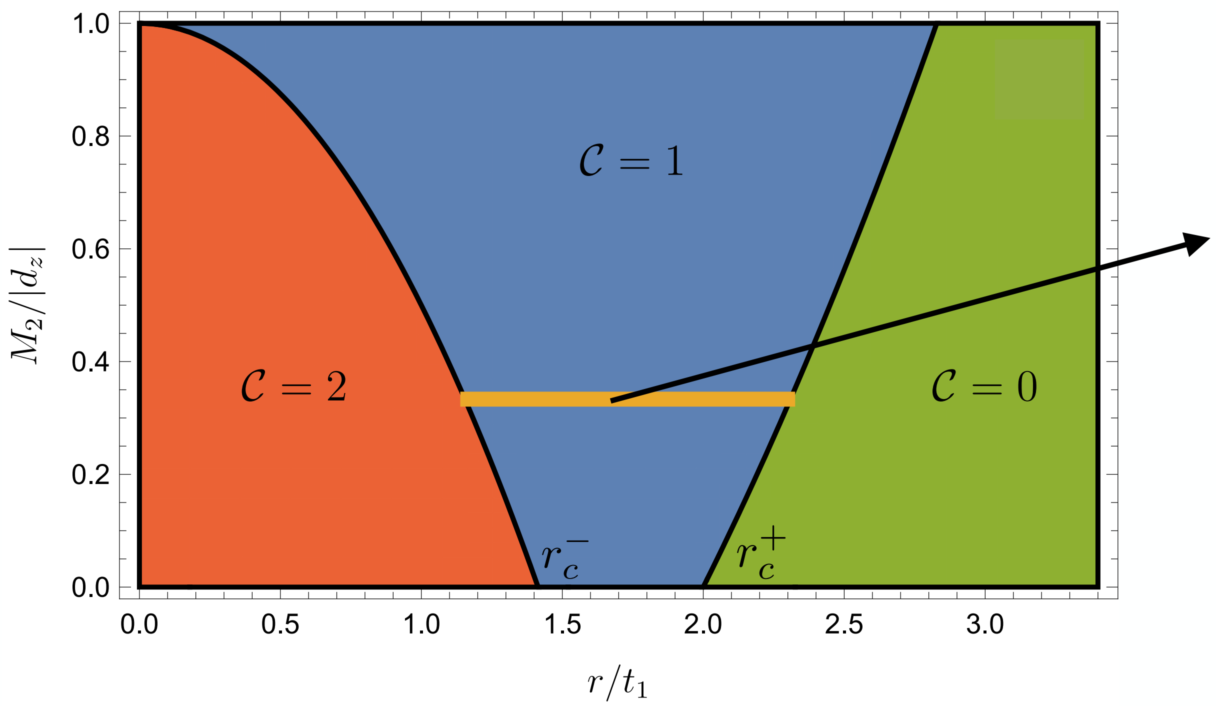


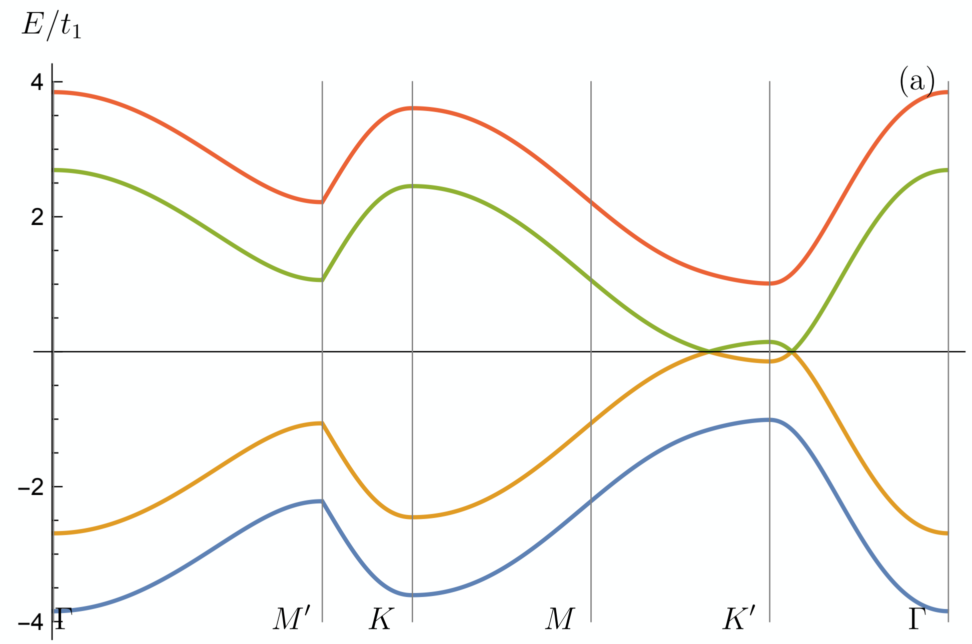
Spheres



Lattice Model

- Is the fractional invariant observable in a lattice model?





- Nodal ring semimetal

Lattice Model

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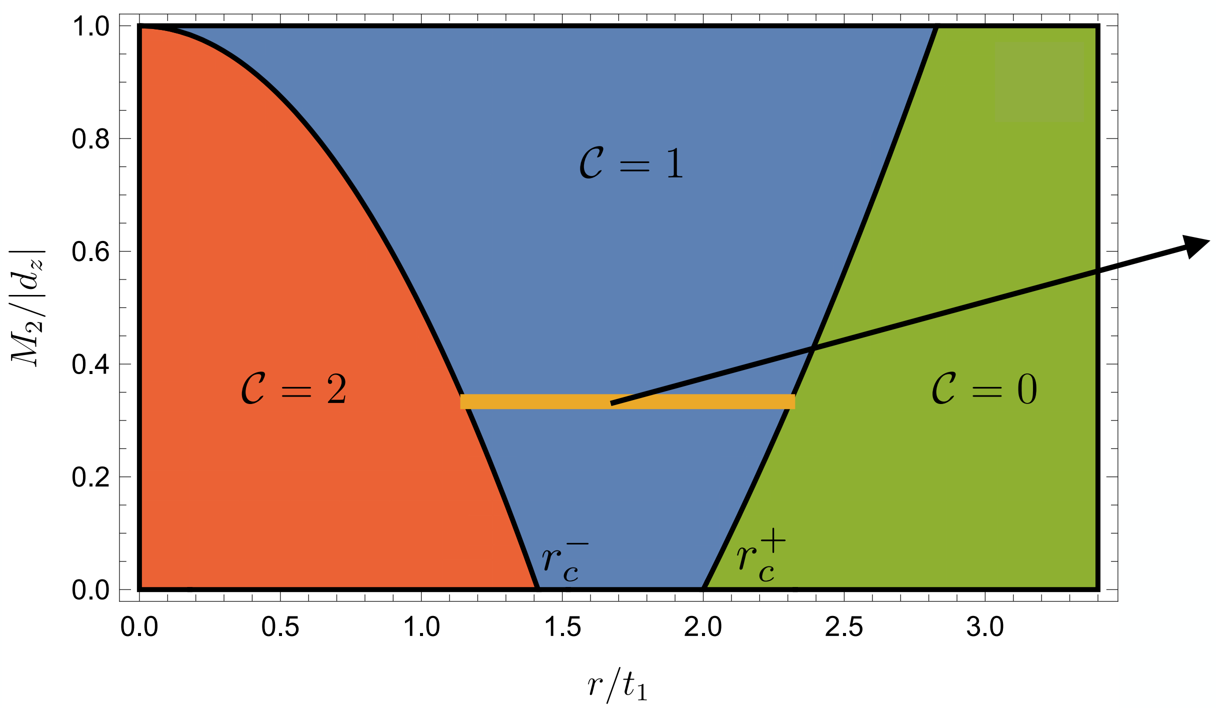

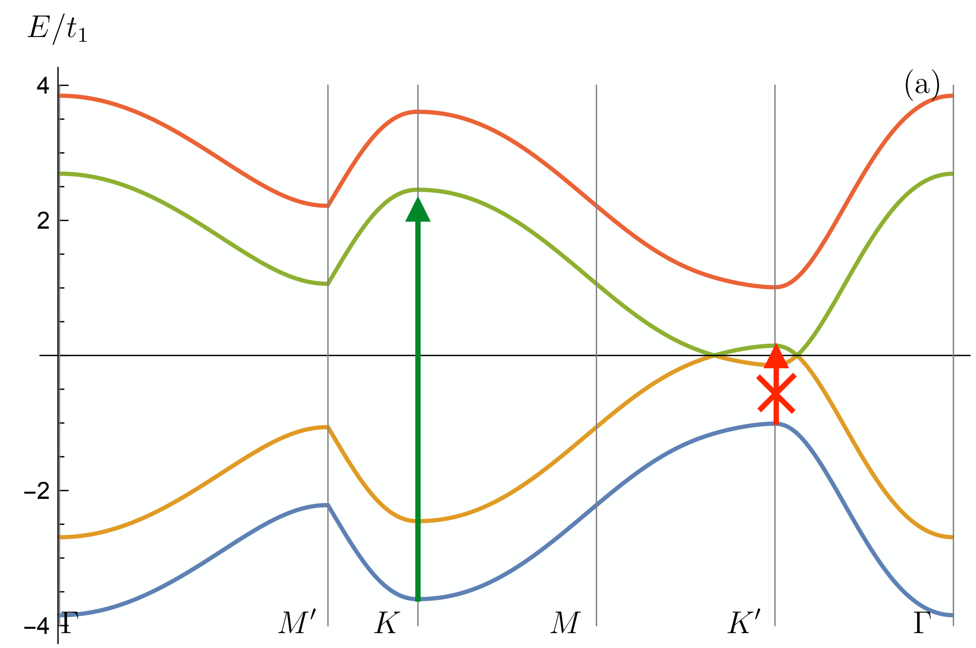

- Nodal ring semimetal
- Circular light transitions forbidden at K'

$$\mathcal{C} \sim \frac{1}{2}\int_0^\infty d\omega \sum_{\boldsymbol{k}=K,K'} \left(\Gamma_l^+(\omega,\boldsymbol{k}) - \Gamma_l^-(\omega,\boldsymbol{k})\right)$$

P. Klein, A. Grushin, and K. Le Hur, arXiv e-prints , arXiv:2002.1742 (2020).

D. T. Tran, A. Dauphin, A. G. Grushin, P. Zoller, and G. N., Sciences Advances **3**, e1701207 (2017).

Lattice Model

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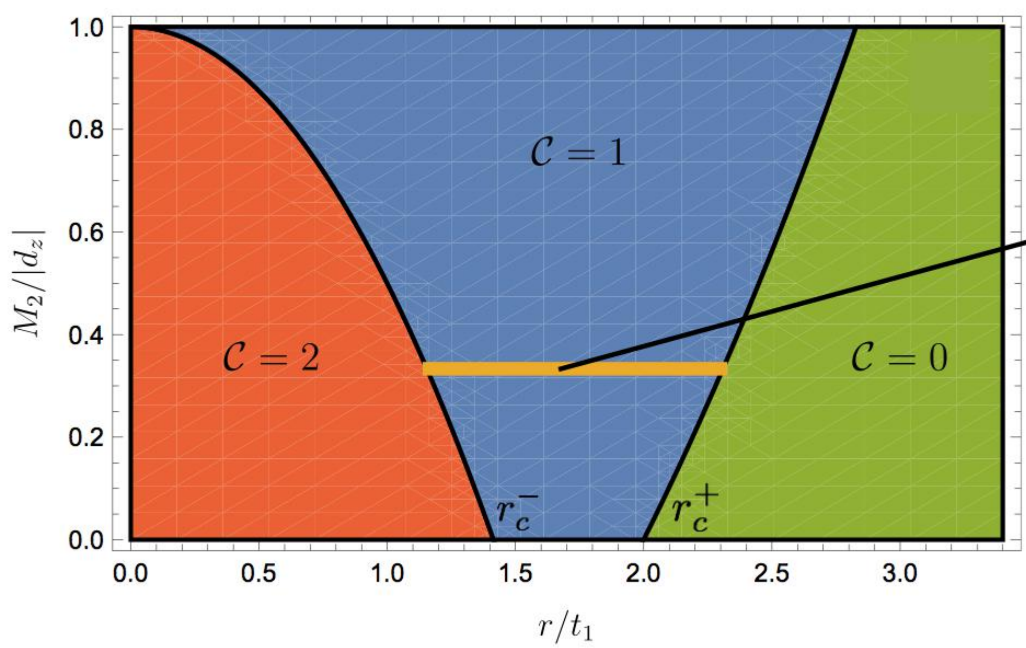
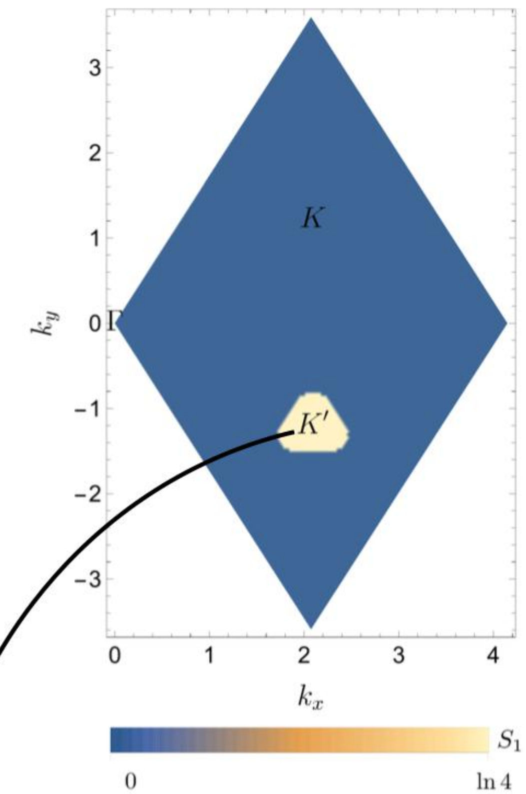




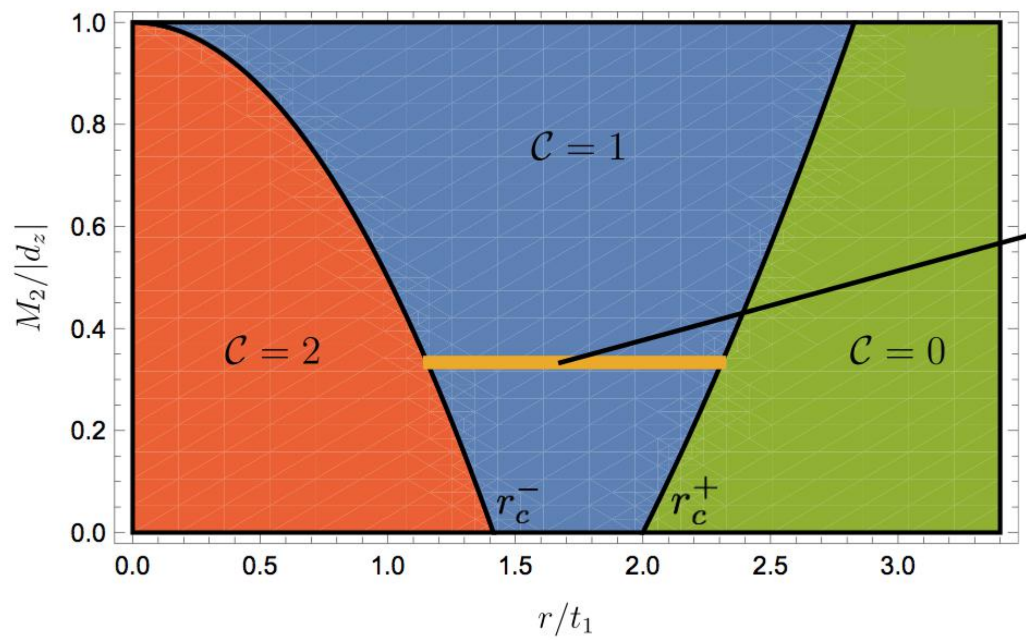
Entanglement entropy

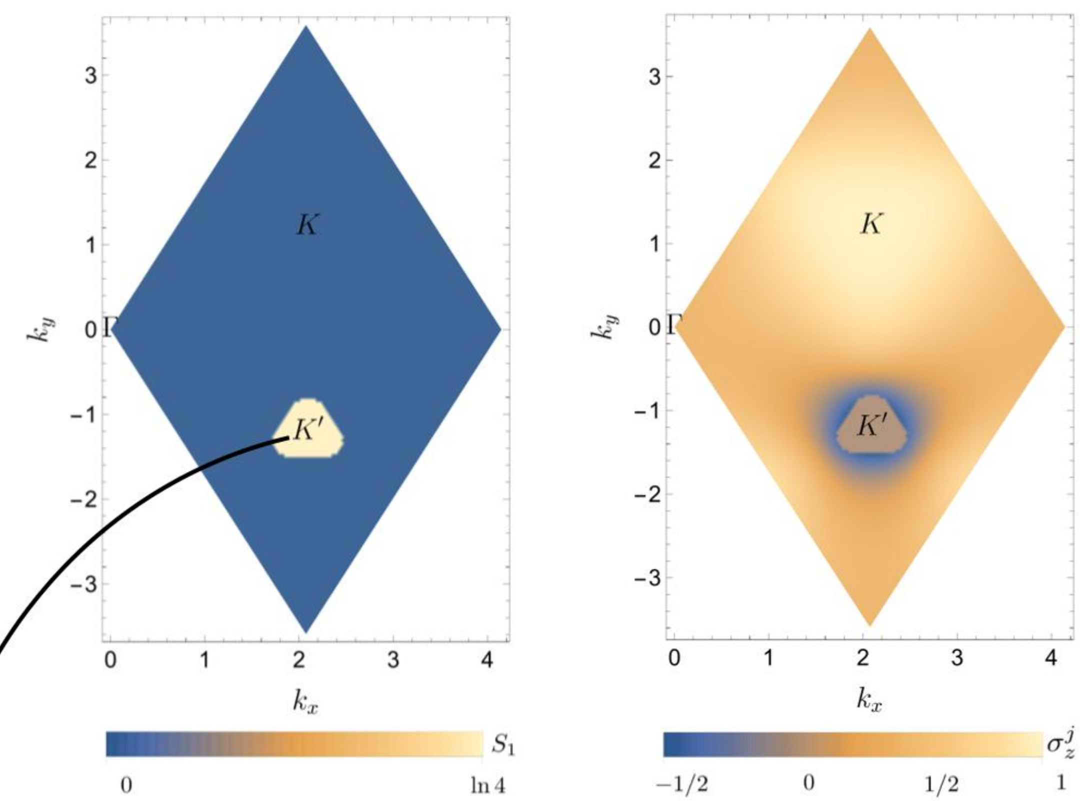
$$S_1 = -\rho_1 \ln \rho_1$$

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

Lattice Model

- Is the fractional invariant observable in a lattice model?





Entanglement entropy

$$S_1 = -\rho_1 \ln \rho_1$$

Sublattice magnetization

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

Lattice Model

- Mapping between 2 qubits and bilayer:

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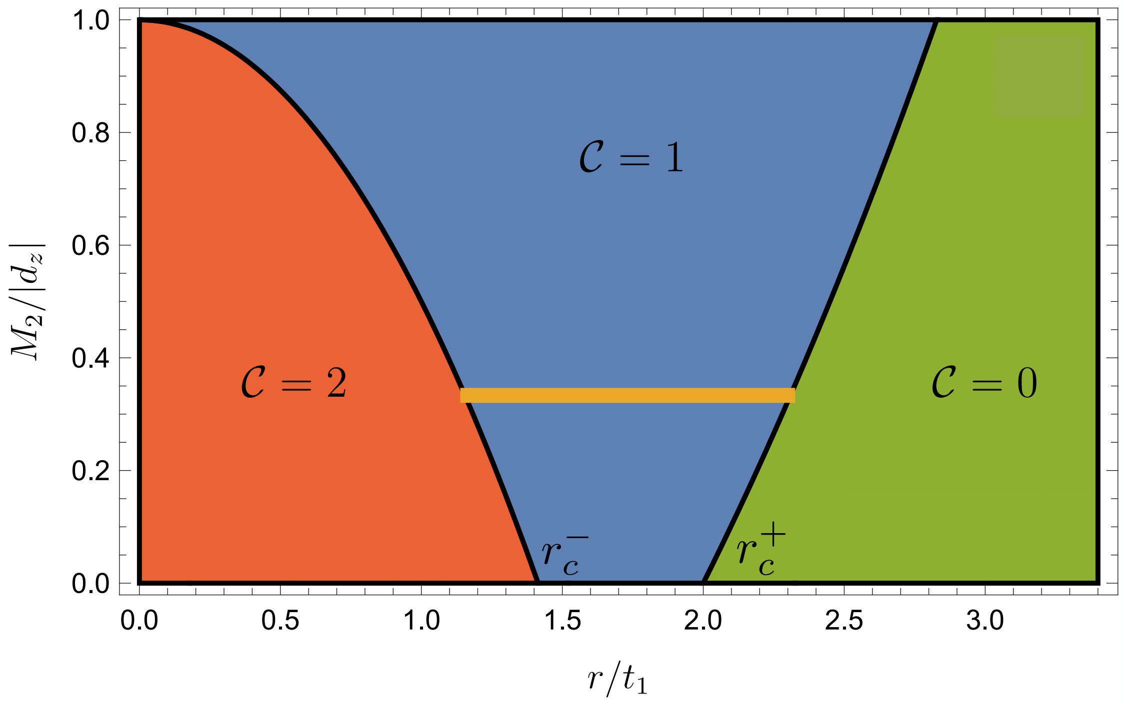
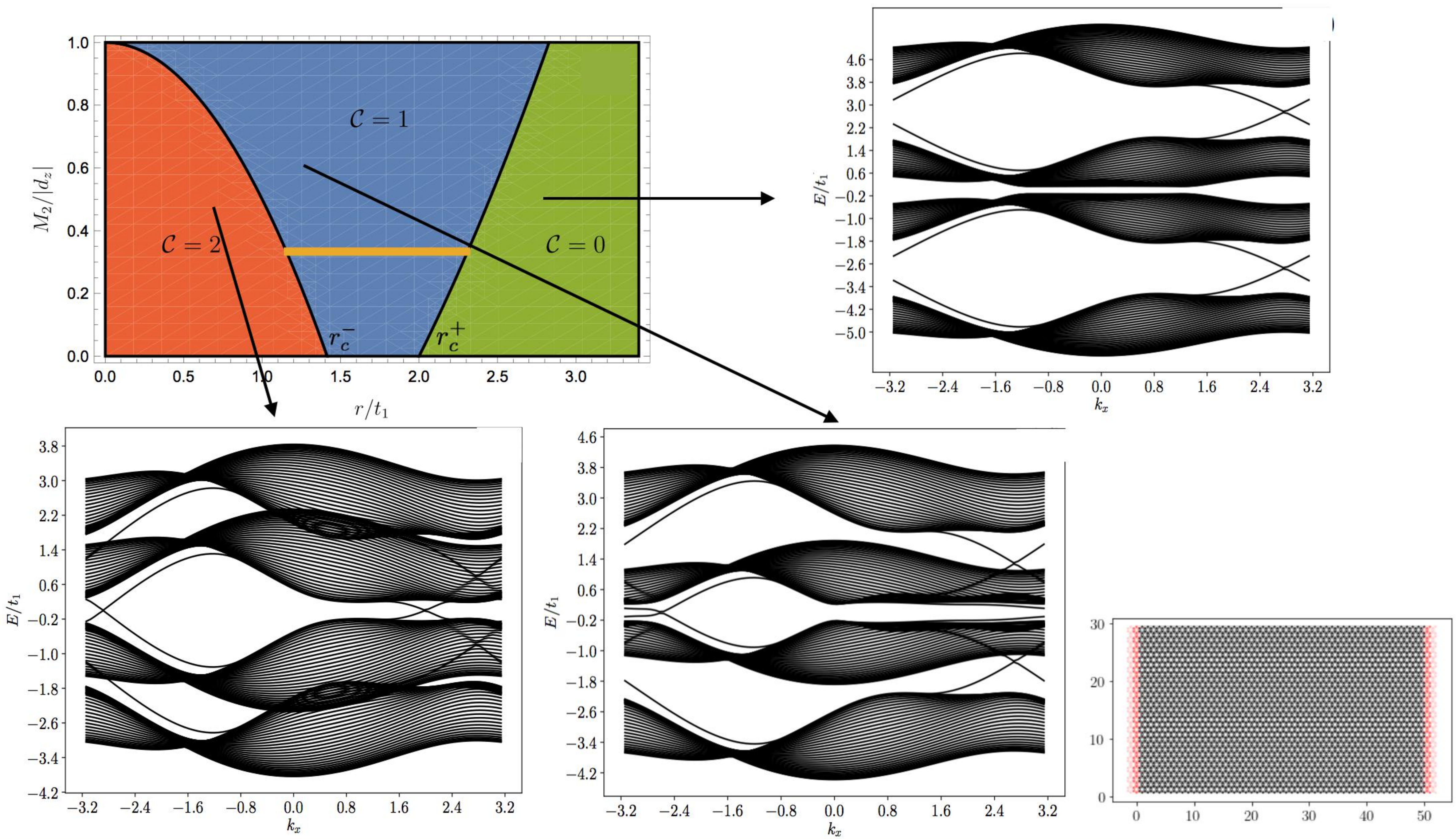


$$\tilde{\mathcal{C}}^j = \frac{1}{2}\langle n^j_{KB} - n^j_{KA} - n^j_{K'B} + n^j_{K'A}\rangle$$
$$= \begin{cases} 1 & r < r_c^- \\ 1/2 & r_c^- < r < r_c^+ \\ 0 & r > r_c^+, \end{cases}$$

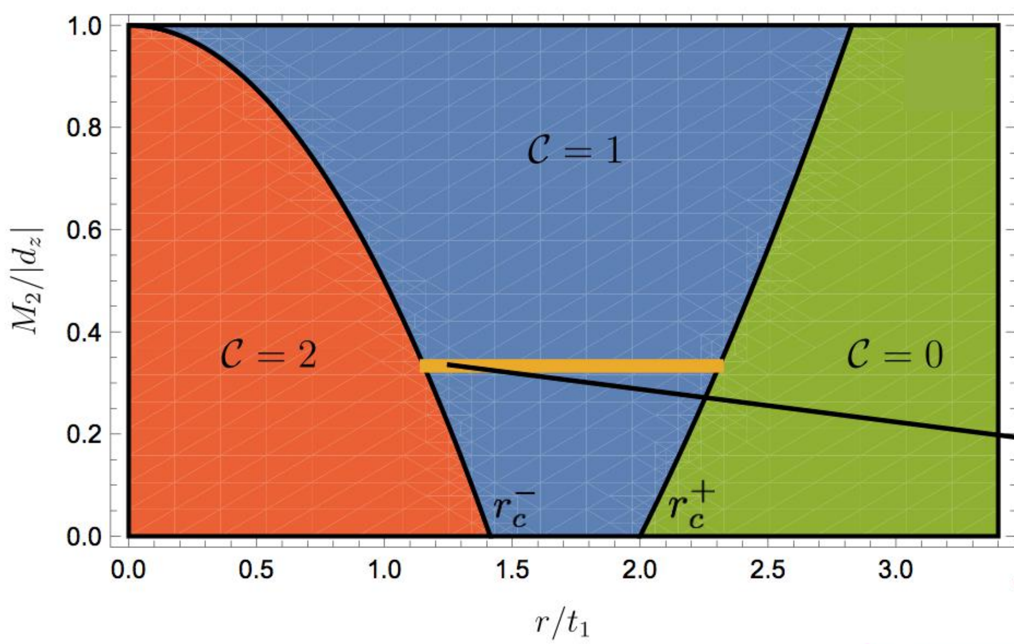
Lattice Model

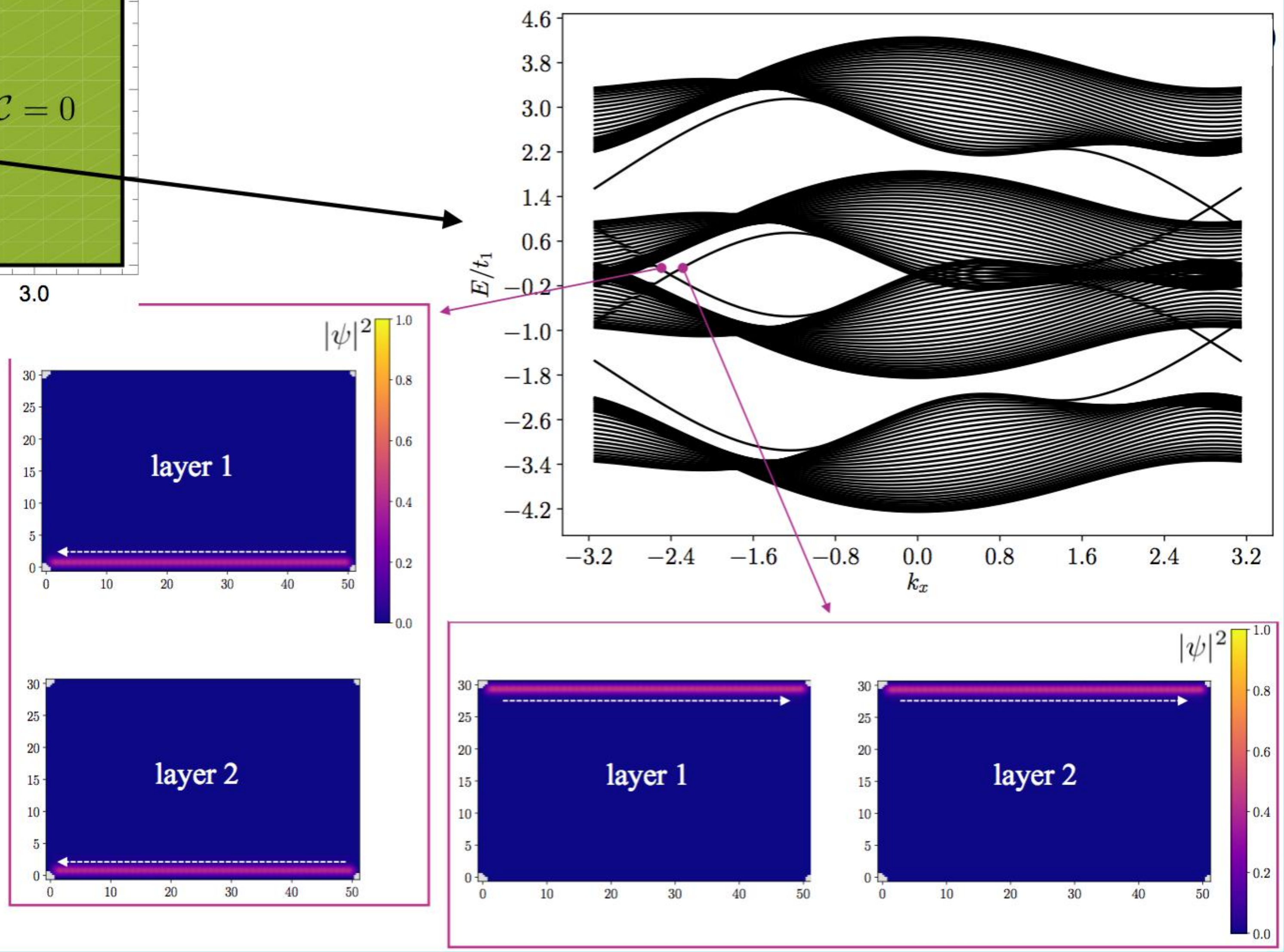
- Edge states in ribbon geometry



Lattice Model

- Edge states in ribbon geometry





- Edge currents split between layers

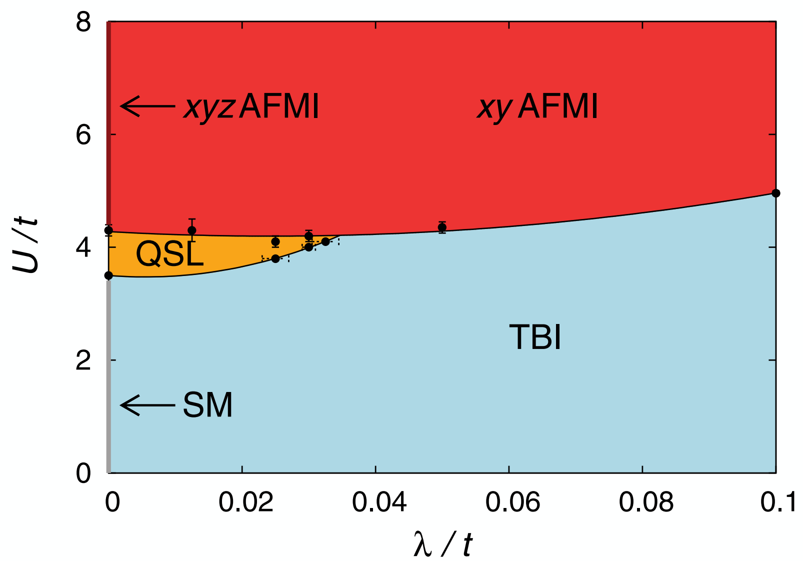
Conclusions

- There is a gauge invariant topological partial Chern number for each spin in interacting models with a radial magnetic field.
- This is rational-valued for models that yield entanglement at one pole.
- Spin models are topologically dual to fermion models on a hexagonal lattice, observable via: sublattice magnetization, entanglement entropy, circler dichroism, edge states.

QMC questions

- Does the nodal ring semimetal survive interactions?

Kane-Mele Hubbard model



PHYSICAL REVIEW B **85**, 115132 (2012)

Quantum phase transitions in the Kane-Mele-Hubbard model

M. Hohenadler,[1] Z. Y. Meng,[2,3] T. C. Lang,[4] S. Wessel,[4] A. Muramatsu,[3] and F. F. Assaad[1]

Semenoff mass

$$\hat{H} = -t\sum_{\langle ij\rangle\sigma}\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + \sum_{i}\left(\frac{1}{2M}\hat{P}_i^2 + \frac{\kappa}{2}\hat{Q}_i^2\right) - g\sum_{i}\hat{Q}_i\hat{\rho}_i.$$

PHYSICAL REVIEW LETTERS **122,** 077601 (2019)

Charge-Density-Wave Transitions of Dirac Fermions Coupled to Phonons

Chuang Chen,[1,2] Xiao Yan Xu,[3] Zi Yang Meng,[1,4,5] and Martin Hohenadler[6]

ALF: Color symmetry

$$\hat{H}_{\lambda} = -\lambda\sum_{\hexagon}\left(\sum_{\langle\langle \boldsymbol{i},\boldsymbol{j}\rangle\rangle\in\hexagon} \mathrm{i}\nu_{\boldsymbol{ij}}\hat{\boldsymbol{c}}^{\dagger}_{\boldsymbol{i}}\boldsymbol{\sigma}\hat{\boldsymbol{c}}_{\boldsymbol{j}} + \mathrm{H.c.}\right)^2$$

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Superconductivity from the condensation of topological defects in a quantum spin-Hall insulator

Yuhai Liu[1], Zhenjiu Wang[2], Toshihiro Sato[2], Martin Hohenadler[2], Chong Wang[3], Wenan Guo[1,4] & Fakher F. Assaad[2]

Thanks for your attention!